Industrial QUALITY CONTROL

Vol. XIII, No. 8
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Simplified Computations for Multiple Regression

The Principles of Experimental Design

QUALITY
through
Statistical Methods

Journal of the
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ASQC Convention & Exposition Chart

11th Annual Convention • Masonic Temple • Detroit, Mich.
May 22-24, 1957

CONVENTION HIGHLIGHTS

TECHNICAL PROGRAM

A program of 90 presentations has been planned with the assistance of the divisions of the Society. Outstanding speakers from the United States and abroad will cover a diversity of subjects on theory and application. Some innovations in the approach to the solution of problems in Automotive, Metals, Textile, Pharmaceutical, Brewing, Chemical and other industries will be presented. Quality Control applications in job shops will also be covered.

DIVISIONAL MEETINGS

The following annual meetings will be preceded by dinners at 6:30 p. m., Wednesday, May 22, at the Sheraton-Cadillac Hotel:

Administrative Applications Division
Aircraft Division
Automotive Division
Chemical Division
Electronics Division
Metals Technical Committee

PLANT TOURS

A series of outstanding plant tours has been arranged for Thursday afternoon, May 23rd. Chartered busses will be available at the Masonic Temple and will return to the downtown area. Since these tours must be restricted to a limited number, tickets must be obtained early at the registration desk. A modest registration fee will be charged to assist in defraying the expenses of this activity. Plant visits will include:

Plymouth Division—Chrysler Corp.
"Automated Engine Plant"
Cadillac Motor Car Div.—GM Corp.
"Final Assembly Plant"
Ford Motor Car Co.
"Rouge Operations"
Parke Davis & Co.
"Manufacturing Laboratories"

SOCIAL NOTES

An informal party is planned for Wednesday evening, May 22nd, in the ballroom of the Sheraton-Cadillac, with music for dancing. Refreshments will be included in the admittance fee. Admission for badge holders only.

FOR THE LADIES . . .

Feminine guests are assured of having a most enjoyable time. A tea, featuring a fashion and hairstyle show, will be given at the Women's City Club of Detroit on Wednesday afternoon, May 22nd. The following day the ladies will visit the historic and world-famous Greenfield Village and be "on their own" for luncheon at Dearborn Inn. Chrysler Corporation's Plymouth Division has extended an invitation for a group to visit their plant Friday morning. A tour of Detroit will also be available. All feminine guests are cordially invited to attend the Wednesday evening party and the annual banquet.

YOU'LL BE INTERESTED TO KNOW THAT . . .

. . . the Book Booth, operated by Wayne University personnel, will display and sell the latest books on Statistical Quality Control, as well as basic texts. These books will cover elementary training, as well as the use of higher mathematics and statistics.

. . . copies of the presentations at the convention will be available there in bound volumes for $3.00 each. Copies may also be obtained by mail for $3.50 by writing to the American Society for Quality Control, Inc., Room 6197 Plankinton Bldg., 161 West Wisconsin Ave., Milwaukee 3, Wis.

. . . special busses will be provided each morning beginning at 8:30 a. m. from the Sheraton-Cadillac and other leading downtown hotels to the Masonic Temple. Beginning at 4:30 p. m., busses will return delegates to the downtown area.

LOOK FOR THESE FEATURES AT THE EXPOSITION

The theme of **The Martin Company** exhibit will be "Research and Development in Quality Control". Their exhibit will portray through pictures, graphs, testing instruments and words the team effort at Martin between Quality Control, Engineering and Manufacturing Research.

Triplett & Barton, Inc. will feature their "Tri-Ind-X" 275 KVP portable, industrial X-ray unit. Adaptable for laboratory, factory and field use, the manufacturer will demonstrate how this product can solve quality control problems.

Included in the **Pratt & Whitney Co., Inc.** exhibit will be an automatic radial play gage, a new design Air-O-Limit gage and an automatic Sigmatic gage. They will also display samples of their cutting tools and Kellerflex products.

Convention members will also be able to inspect **IBM's** new Automatic Production Recording System which is a fully-integrated system designed specifically for the automatic collection of production data at the source of manufacturing operations, for quick action by management.

The exhibit area will be adjacent to the meeting rooms in the Masonic Temple. Don't miss this opportunity to see what's new in quality control equipment and learn how other companies are handling their quality control problems.

For space and information on the exposition, contact:
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1849 W. 24 St. - Cleveland 13, Ohio
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From the President's Desk

Some Thoughts on Education

Someone with the gift for pointed satire has coined the very apt reminder that "It is what a man learns after he knows it all that really counts." Which is simply a neat way of saying that this process of "education" must of necessity be a continuing experience if it is to nurture the development of a capacity for greater satisfaction.

This matter of "education," whether looked at from the formal or informal point of view is, in my judgment, the bed-rock on which all else in ASQC rests. It is its *raison d'être*. As a matter of history, the origins of ASQC trace back to the well-known eight-day training courses under WPB during World War II. The Constitution of the Society spells out as a specific purpose of ASQC "the advancement and diffusion of knowledge." The whole pattern of our formal programs, whether at the Section, Region, or National level is built around this theme.

Perhaps this seems to be laboring an obvious point, but in a very real way, I think not. It is easy to confuse the *fruits* of education with the *experience* of education itself. We see in the spectacular growth of ASQC ample evidence of the *fruits* of this "diffusion of knowledge" which has been the driving force behind Society effort. We become understandably enthusiastic about the tangible results that often flow from the techniques we have learned to put into practice in the field of industrial quality control. We witness the spectacle of advancement and widening horizons of opportunity for many in the SQC field who have in various ways pursued the study and application of this science.

Yet these are but the *fruits* of education. It is the *experience* of education itself that excites my imagination. Education, as I see it, is not just the acquisition of facts and more facts; not just a matter of acquiring familiarity with formulas and techniques and all the necessary and impressive structural elements of the science of quality control—or, for that matter, any other discipline. Education is an intangible, personal thing—a matter of spiritual, intellectual, inner satisfaction—an enriching experience that includes much more than the storing up of information. Machines exist for storing up information and for performing mechanically feats of "mental" gymnastics far surpassing anything that human skill can even faintly duplicate. But a machine cannot experience the subtle alchemy of education. It never knows the stimulating satisfaction of achievement; it cannot respond to ideas; it cannot know the exhilaration of imagination, or react to the urge for creative effort in the quest for a better way. Only the human spirit of man—not just the mind alone—is geared to experience this fascinating mystery of "education." Strip these gears, and man becomes little more than a human robot.

So I submit the thesis that the existence of ASQC, its present stature, and its future destiny, rest squarely on the opportunity it provides as a medium for contributing in many vital ways to the mysterious, intangible, personal phenomenon of "education." The objective of our effort in the quality control field, reduced to essential fundamentals, must not be emphasis on the *fruits* of education, but on the *experience* of education itself. The learning process never stops in human experience, but it can be short-circuited or chanelled up blind alleys. So long as we continue individually and organizationally to emphasize the dignity and human worth of the challenge SQC presents to man's instincts for exploring new areas of educational adventure, we shall be on safe ground.

Dale L. Lobsinger

Average Sample Number Under Curtailed or Truncated Sampling

IRVING W. BURR

Purdue University, West Lafayette, Ind.

The Problem

Four kinds of sampling for attributes are in common use:

(a) Single sampling

(b) Double sampling

(c) Multiple sampling (sometimes called group sequential)

(d) True sequential sampling

Comparisons of such plans are often made on the basis of the amount of inspection they require, that is, on the average sample number, or ASN. Now of course a comparison of ASN's is only fair if the plans in question provide the same degree of protection, that is, have the same operating characteristic or OC curves.

Calculation of the ASN for single and double sampling plans are explained in many books, see, for example, Ref. (1) and (2). Calculation of the ASN for a multiple sampling plan is not different in principle from that for double sampling, but is far longer, owing to the much larger number of distinct ways in which acceptance can occur. Thus we may have to go through to the maximum number of samples (perhaps eight) before accepting. On the other hand calculation of the ASN for pure sequential sampling is simple, from formulas given in Ref. (3). Fortunately the OC curves for many multiple sampling plans are given in Ref. (4).

In order to obtain more favorable ASN curves, while still obtaining the same protection (OC curve), inspectors often resort to "curtailed" or "truncated" sampling. Thus as soon as a decision for acceptance or rejection is certain, we may stop inspection and take the indicated action. In this way we may be able to get a considerably smaller ASN, especially if the quality is poor, while still having exactly the same OC curve.

We might use curtailed sampling on single, double or multiple sampling, but not on sequential sampling, because it is already perfectly curtailed, being piece by piece. Now ordinarily we never use curtailing on single sampling, nor on the first sample of double and multiple, because we wish to complete a uniform size of sample for the record of past performance. On the second sample of double sampling we find the greatest use of curtailed sampling. We therefore want to develop the method for calculating the ASN curve for double sampling with curtailing in the second sample. The method is also applicable to other cases. Although we can and should curtail sampling after the first sample in multiple sampling, it is scarcely feasible to calculate the ASN for such plans.

Example Showing the Method

Suppose we have given the double sampling plan

$$n_1 = 50 \qquad n_2 = 100$$
$$c_1 = 0 \qquad c_2 = 2$$

If on the first sample we have two defectives, then any defective in the second sample rejects the lot. If this should occur on the twelfth piece, then the sample size on a curtailed basis is 62, while on the uncurtailed basis it would be 150, as it always is, whenever a second sample is called for. The usual ASN for double sampling is an average of 50's and 150's, weighted by the proportions of times we get a decision on the first sample or the second sample respectively. The curtailed sampling ASN is an average of 50's and numbers running from 51 to 150, and is almost always smaller than the usual ASN.

In what follows we shall only curtail when rejection is certain and not when acceptance occurs. Thus, if we find one defective among the first 50, we take a second sample of 100. Suppose we find no more defectives. Then after 99 of the 100 pieces are inspected we could confidently accept, because even if the 100th piece were defective we would only have a total of two defectives, which is still permissible for acceptance. Since the savings between 149 and 150 are so negligible in comparison with those obtainable under rejection, we neglect this possibility in the calculation.

We shall assume that the lot size is large enough for us to use the binomial distribution in figuring the probabilities. Then on the first sample from a lot with fraction defective, $p' = 0.015$, we have

$$P \text{ (no defective in 5)} = 0.985^{50} = 0.46969^*$$

$$P \text{ (one defective in 50)} = 50(0.985)^{49}\,(0.015) = 0.35763$$

$$P \text{ (two defectives in 50)} = \frac{50(49)}{2}\,(0.985)^{48}\,(0.015)^2 = 0.13343$$

$$P \text{ (two or less defectives in 50)} = 0.96075$$

$$P \text{ (three or more defectives in 50)} = 1.00000 - 0.96075 = 0.03925$$

Thus on the first sample the probability of immediate acceptance is 0.46969, and of immediate rejection is 0.03925, while the probability of no decision on the first sample is 0.35763 + 0.13343 or 0.49106.

Now under curtailed sampling, how many pieces of the second sample must we expect to take before reaching a decision? There are of course two cases, that is, we may go to the second sample with either one or two defectives against the lot. In the former case we can afford one more defective before rejection, while in the latter we cannot have any more defectives if we are to accept. To find the average sample size we make use of the following formula, the derivation of which will be given in the next section. It was first published in Ref. (2), page 313.

$$\text{ASN} = (k/p')P(k+1 \text{ or more defectives in } n+1) + nP(k-1 \text{ or fewer defectives in } n), \qquad (1)$$

where k defectives in this sample of n pieces would call for rejection, while $k-1$ would not, p' is the true fraction defective, and the probabilities are from the binomial distribution.

*Seven-place logarithms are needed to give a five-place result here.

Now of, say, 100,000 lots sent us at exactly $p' = 0.015$, 35,763 stand to enter the second sample with one defective and 13,343 with two defectives. How many additional pieces will we average in each case? For the *additional* pieces, we have respectively ($k = 2$ and 1):

$$\begin{aligned}\text{ASN (2 will rej.)} &= (2/0.015)\ P(3 \text{ or more in } 101) \\ &\quad + 100P(1 \text{ or less in } 100) \\ &= 133.33[1 - P(2 \text{ or less in } 101)] \\ &\quad + 100P(0 \text{ or } 1 \text{ in } 100) \\ &= 133.33(0.19400) + 100(0.55656) \\ &= 81.52\end{aligned}$$

$$\begin{aligned}\text{ASN (1 will rej.)} &= (1/0.015)\ P(2 \text{ or more in } 101) \\ &\quad + 100P(0 \text{ in } 100) \\ &= 66.67(0.44848) + 100(0.22061) \\ &= 51.96\end{aligned}$$

Hence, since there stand to be 46,969 lots accepted and 3925 lots rejected on the first sample, we have the over-all ASN for truncated sampling, as the average for all possibilities:

$$\begin{aligned}\text{ASN} &= 0.46969(50) + 0.03925(50) + 0.35763(50 + 81.52) + 0.13343(50 + 51.96) \\ &= 50 + 0.35763(81.52) + 0.13343(51.96) \\ &= 86.1\end{aligned}$$

For ordinary sampling we would have

$$\begin{aligned}\text{ASN} &= (0.46969 + 0.03925)50 + (0.35763 + 0.13343)150 \\ &= 99.1\end{aligned}$$

Table I lists the ASN's for double sampling with and without curtailing and for single sampling. The preceding calculations were obtained using the binomial, but since the p' values were small and the n's sizable we could have used the Poisson distribution satisfactorily for the various probabilities.

Thus for our example we could use Table H in reference (2) and find the following:

$n = 50 \qquad np' = 50(0.0015) = 0.75$

P(2 def. or less)	= 0.959	
P(1 def. or less)	= 0.827	Binomial:
P(0 def.)	= 0.472	(vs. 0.46969)
P(2 def.)	= 0.959 − 0.827	
	= 0.132	(vs. 0.13343)
P(1 def.)	= 0.827 − 0.472	
	= 0.355	(vs. 0.35763)

$n = 100 \qquad np' = 100(0.0015) = 1.50$

P(1 def. or less)	= 0.558	(vs. 0.55656)
P(0 def.)	= 0.223	(vs. 0.22061)

$n = 101 \qquad np' = 101(0.0015) = 1.515$

P(2 def. or less)	= 0.805	
P(1 def. or less)	= 0.553	
P(3 def. or more)	= 1 − 0.805	
	= 0.195	(vs. 0.19400)
P(2 def. or more)	= 1 − 0.553	
	= 0.447	(vs. 0.44848)

TABLE I—Average Sample Numbers for Sampling Plans with Like OC Curves

Lot Fraction p'	Double: $n_1 = 50$, $c_1 = 0$; $n_2 = 100$, $c_2 = 2$		Single: $n = 75$, $c = 1$
	Ordinary	Curtailed	
0.000	50	50	75
0.005	72.0	70.8	75
0.010	88.1	82.2	75
0.015	99.1	86.1	75
0.020	105.7	85.3	75
0.025	108.9	82.0	75
0.030	109.3	77.9	75
0.040	104.7	69.6	75

Then the average number of pieces in the second sample, following one defective in the first is

$$\begin{aligned}\text{ASN(if 1 def. on 1st)} &= (2/0.015)(0.195) + 100(0.558) = 81.8 \quad \text{(vs. 81.5)} \\ \text{ASN(if 2 def. on 1st)} &= (1/0.015)(0.447) + 100(0.223) = 52.1 \quad \text{(vs. 52.0)},\end{aligned}$$

while the over-all ASN is

$$\text{ASN} = 50 + 0.355(81.8) + 0.132(52.1) = 85.9 \quad \text{(vs. 86.1)}$$

The largest difference between ASN's using the binomial and Poisson distributions was only 0.3 in the present example.

Figure 1 compares the ASN's for the plans given in Table I with that for the sequential plan

$p'_1 = 0.005 \qquad \alpha = 0.05$
$p'_2 = 0.050 \qquad \beta = 0.10$

for which

$h_1 = 0.9585 \qquad s = 0.0197$
$h_2 = 1.2305$

These plans all have substantially identical OC curves, as does the multiple plan: [4]

Cumulative Sample	*Acceptance Number*	*Rejection Number*
20	*	2
40	*	2
60	0	2
80	1	3
100	1	3
120	1	3
140	2	3

The ASN curve for this multiple plan, if curtailed would probably be between the curves for pure sequential and curtailed double sampling, except right at the beginning. Figure 1 clearly shows the relatively large gain through curtailing, especially when quality is poor.

If the lot size is not sufficiently large to use the binomial, a considerably more complicated technique would need to be used.[5]

Derivation

The proof of the main formula (1) is somewhat difficult and does require some calculus.

In what follows we shall assume the binomial distribution. Moreover we shall never truncate in acceptance, that is, even when acceptance has already become sure, we shall still look at all n pieces. Further, the number of pieces to be inspected is n, the rejection number is k (that is, k defectives calls for rejection, but $k - 1$ gives acceptance), and the fraction defective is p' and $1 - p' = q'$.

There are the two possible outcomes: acceptance and rejection. In the latter case under truncation we will always have k and only k defectives. The probability of the first k pieces being defective is $(p')^k$. Hence the contribution to the ASN in this event is $k\,(p')^k$. Next suppose that the k-th defective occurs on the $(k + 1)$st case. The probability of $k - 1$ defectives in the first k pieces, and then of a defective on the $(k + 1)$st piece is the product of a binomial probability and p', that is,

$$[C(k, k - 1)p'^{k-1}q']p'$$

Similarly if the k-th defective occurs on the $(k + j)$th piece the probability is

$$[C(k + j - 1, k - 1)p'^{k-1}q'^{j}]p',$$

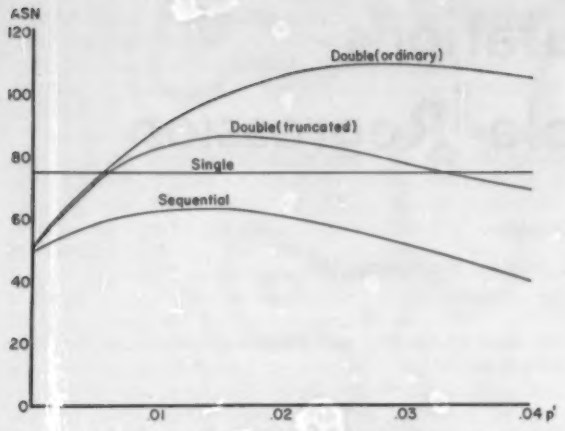

Figure 1—ASN Curves for 4 Sampling Plans Having Same OC Curves ($p'_1 = 0.005$, $\alpha = 0.05$; $p'_2 = 0.050$, $\beta = 0.10$)

and the contribution to ASN in this case is

$$(k + j)[C(k + j - 1, k - 1)p'^{k-1}q'^{j}]p'$$

$$= (k + j)\frac{(k + j - 1)!p'^{k}q'^{j}}{(k - 1)!j!}$$

$$= \frac{(k + j)(k + j - 1)\dots(j + 1)p'^{k}q'^{j}}{(k - 1)!} \quad (2)$$

On the other hand acceptance may occur with anywhere from zero to $k - 1$ defectives among the n pieces. The probabilities of there being exactly i defectives in the n pieces is, by the binomial distribution,

$$C(n,i)p'^{i}q'^{n-i} \quad (3)$$

the cases included being with $i = 0$ to $k - 1$. The contribution to the ASN is obtained by multiplying each such term in (3) by n.

Thus we have altogether

$$ASN = \sum_{j=0}^{n-k} \frac{(k + j)(k + j - 1)\dots(j + 1)}{(k - 1)!} p'^{k}q'^{j}$$

$$+ n \sum_{i=0}^{k-1} C(n,i)p'^{i}q'^{n-i} \quad (4)$$

The latter is merely n times the binomial probability of $k - 1$ defectives or less in n. To evaluate the former summation we write it as

$$\frac{p'^{k}}{(k - 1)!} \sum_{j=0}^{n-k} (k + j)(k + j - 1)\dots(j + 1)q'^{j} \quad (5)$$

We shall want to simplify the summation in (5), into a tabulated distribution.

$$\sum_{j=0}^{n-k} (k + j)(k + j - 1)\dots(j + 1)q'^{j}$$

$$= \sum_{j=0}^{n-k} \left(\frac{\partial^k}{\partial q'^{k}} q'^{k+j} \right)$$

$$= \frac{\partial^k}{\partial q'^{k}} \left(\sum_{j=0}^{n-k} q'^{j} \right)$$

$$= \frac{\partial^k}{\partial q'^{k}} \left(\sum_{j=k}^{n} q'^{j} \right)$$

$$= \frac{\partial^k}{\partial q'^{k}} \left(\sum_{j=0}^{n} q'^{j} \right) \text{ since the k-th derivatives of terms 1 to } q'^{k-1} \text{ are zero} \quad (6)$$

$$= \frac{\partial^k}{\partial q'^{k}} \left[\frac{1 - q'^{n+1}}{1 - q'} \right] \text{sum of a geometric progression} \quad (7)$$

Now for the latter we use Leibnitz' theorem for the continued derivative of a product:

$$\frac{d^k}{dx^k} f(x)g(x)$$

$$= \sum_{i=0}^{k} C(k,i) \left[\frac{d^i}{dx^i} f(x) \right] \left[\frac{d^{k-i}}{dx^{k-i}} g(x) \right] \quad (8)$$

Hence (7) becomes

$$\sum_{i=0}^{k} C(k,i) \frac{\partial^i}{\partial q'^{i}} (1 - q'^{n+1}) \frac{\partial^{k-i}}{\partial q'^{k-i}} (1 - q')^{-1}$$

$$= (1 - q'^{n+1})k!(1 - q')^{-k-1}$$

$$+ \sum_{i=1}^{k} \frac{k!}{i!(k-i)!} (-1)(n+1)(n)$$

$$\dots(n-i+2)q'^{n-i+1} \quad (k-i)!(1-q')^{-k+i-1}$$

$$= (1 - q'^{n+1})k!p'^{-k-1}$$

$$- \sum_{i=1}^{k} \frac{k!}{i!} \frac{(n+1)!}{(n-i+1)!} \quad q'^{n-i+1}p'^{-k+i-1}$$

$$= k!\, p'^{-k-1}[1 - q'^{n+1} - \sum_{i=1}^{k} \frac{(n+1)!\, q'^{n+1-i}p'^{i}}{i!\,(n-i+1)!}]$$

$$= k!\, p'^{-k-1}[1 - P(k \text{ or less def. in } n + 1)] \quad (9)$$

Replacing the summation in (5) by the last expression we have

$$(k/p')[1 - P(k \text{ or less def. in } n + 1)]$$

Finally we have therefore

$$ASN = (k/p')\, P(k + 1 \text{ or more def. in } n + 1) + nP(k - 1 \text{ def. or less in } n), \quad (10)$$

where the probabilities are binomial.

Some effort was made to derive (1) as a specialization and limit of results in reference (5), but the connection if any was not established.

References

(1) Grant, E. L., *Statistical Quality Control*, McGraw-Hill Book Co., New York, 1952

(2) Burr, I. W., *Engineering Statistics and Quality Control*, McGraw-Hill Book Co., New York, 1953

(3) Statistical Research Group, *Sequential Analysis of Statistical Data: Applications*, Columbia University Press, New York, 1945

(4) *Military Standard Sampling Procedures and Tables for Inspection by Attributes*, MIL—STD—105A, U. S. Government Printing Office

(5) Vajda S., "Average Sample Numbers from Finite Lots," *Supplement to Journal of Royal Statistical Society*, vol. 8 (no. 8, part 2, 1946) p. 198-201

Simplified Computations for Multiple Regression

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Introduction and Summary

Most advanced and many elementary statistical textbooks consider multiple regression problems both from a theoretical and a computational point of view. This article will consider only the computational method of multiple regression.

Many ideas for the following computational method were obtained from Anderson and Bancroft[1], Duncan and Kenny[2], and Peach[3]. It is hoped that this elementary treatment of the subject with its illustrative example will aid the research worker in analyzing multiple regression problems with the least effort and time. This method is suitable for use with any fully automatic desk calculating machine.

The main advantages of this method are:

(a) the numbers of digits to the left of the decimal points of the elements of the sums of squares and sums of products matrix are adjusted to be one or zero which permits the use of a uniform number of decimal places in the calculations;

(b) the multiple correlation coefficient and entries for the analysis of variance table for multiple regression can be found without computing the regression coefficients and the inverse of the sums of squares and sums of products matrix;

(c) the additional reduction due to any regression variable over that obtained for previous ones is obtainable for every regression variable;

(d) the research worker can fit only those regression variables that add a significant additional reduction if he so desires;

(e) time or work is not lost if one wishes to obtain the regression coefficients and their variances; and

(f) numerous checks are employed on the calculations that are required.

The following method is especially suited for a research problem which has several dependent variables with one set of independent variables. It allows the worker to decide which dependent variables are explained by regression with the least time and work. Regression coefficients and their variances are not usually wanted unless the regression is significant. This procedure eliminates the need of caculating these quantities when prediction is not good enough to be useful.

Algebraic Procedure

For simplicity, this method will be illustrated by considering four independent variables (x_1, x_2, x_3, x_4), and one dependent variable, (y). First compute and record the sums of squares and sums of products in the following manner:

$$(1) \qquad \begin{matrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{1y} \\ & a_{22} & a_{23} & a_{24} & a_{2y} \\ & & a_{33} & a_{34} & a_{3y} \\ & & & a_{44} & a_{4y} \\ & & & & a_{yy} \end{matrix}$$

where

$$a_{ii} = \sum_{\alpha=1}^{n} x_{i\alpha}^2 - \frac{\left(\sum_{\alpha=1}^{n} x_{i\alpha}\right)^2}{n},$$

$$a_{ij} = \sum_{\alpha=1}^{n} x_{i\alpha} x_{j\alpha} - \frac{\left(\sum_{\alpha=1}^{n} x_{i\alpha}\right)\left(\sum_{\alpha=1}^{n} x_{j\alpha}\right)}{n},$$

$$a_{iy} = \sum_{\alpha=1}^{n} x_{i\alpha} y_{\alpha} - \frac{\left(\sum_{\alpha=1}^{n} x_{i\alpha}\right)\left(\sum_{\alpha=1}^{n} y_{\alpha}\right)}{n},$$

$$a_{yy} = \sum_{\alpha=1}^{n} y_{\alpha}^2 - \frac{\left(\sum_{\alpha=1}^{n} y_{\alpha}\right)^2}{n},$$

$i = j = 1, 2, 3, 4$, and n is the number of observations.

The first feature of this method is that the sum of squares for the dependent variable, a_{yy}, is recorded as the last entry of the column containing the sums of products of the dependent variable with the independent variables. The addition of a_{yy} to the last column results in a square matrix. This feature will be utilized to adjust the number of digits preceding the decimal points in the elements of the above matrix. The residual sum of squares is also obtained directly by adding the term a_{yy} to the last column of (1).

Then*, in order to simplify the calculations, make the diagonal terms (a_{11}, a_{22}, a_{33}, a_{44}, a_{yy}) lie between 0.1 and 10 by pre- and post-multiplying (1) by a diagonal matrix of powers of ten which is as follows:

$$(2) \qquad \begin{bmatrix} 10^{q_1} & 0 & 0 & 0 & 0 \\ 0 & 10^{q_2} & 0 & 0 & 0 \\ 0 & 0 & 10^{q_3} & 0 & 0 \\ 0 & 0 & 0 & 10^{q_4} & 0 \\ 0 & 0 & 0 & 0 & 10^{p} \end{bmatrix}$$

* The reader will find it very helpful at this point to match each step of the author's general exposition with its numerical counterpart in the "Illustrative Example" beginning on page 10.

This will also result in the sums of products having at most one digit before the decimal points, thus allowing a uniform number of decimal places in all future calculations. The values of the q_i's and p are determined as follows:

Consider only a diagonal term and in it the largest *even* number of places through which the decimal point must be shifted (left or right) to make that term be a number between 0.1 and 10. Then divide this even number by two to get the applicable value of q_1 or p. For example, if $a_{11} = 8{,}238.93$, q_1 would be -2; if $a_{33} = 2{,}213{,}922.00$, q_3 would be -3; and if $a_{yy} = 5{,}098.35$, p would be -2. The above method is an extension of an idea formulated by Duncan and Kenney[2].

After pre- and post-multiplying (1) by (2), which in effect is accomplished by adding the q_i's and p according to the term we are adjusting and shifting the decimal point the number of places indicated by the sum, we obtain a matrix of a^*'s. If, as in the above paragraph, $q_1 = -2$ and $p = -2$, we would shift the decimal point of a_{11} four places to the left and the decimal point of a_{1y} four places to the left, etc.

The work sheet will look as follows:

$$(3)\qquad \begin{array}{ccccc} a^*_{11} & a^*_{12} & a^*_{13} & a^*_{14} & a^*_{1y} \\ & a^*_{22} & a^*_{23} & a^*_{24} & a^*_{2y} \\ & & a^*_{33} & a^*_{34} & a^*_{3y} \\ & & & a^*_{44} & a^*_{4y} \\ & & & & a^*_{yy} \end{array}$$

where $a^*_{ii} = 10^{2q_i}a_{ii}$, $a^*_{ij} = 10^{(q_i+q_j)}\,a_{ij}$; $a^*_{iy} = 10^{(q_i+p)}\,a_{iy}$, and $a^*_{yy} = 10^{2p}a_{yy}$. The matrix (3) is then reduced by the Abbreviated Doolittle Method after calculating a^*_{ic}, which is the sum of the terms in the i^{th} row *including* the terms in the i^{th} row omitted because of symmetry. The column of a^*_{ic}'s is used to provide checks on the calculations required to reduce (3).

Since a_{yy} is the total sum of squares used in an analysis of variance table for a multiple regression problem, when we reduce (3) by the Abbreviated Doolittle Method, the term resulting from a^*_{yy} is used to compute the residual sum of squares after fitting all the x_i's. In fact, this sum of squares is readily calculated by multiplying the term resulting from a^*_{yy} by 10^{-2p}.

Algebraic Procedure for the Forward Solution of the Abbreviated Doolittle Method

The work sheet should now look like:

$$(4)\qquad \begin{array}{ccccc|c} a^*_{11} & a^*_{12} & a^*_{13} & a^*_{14} & a^*_{1y} & a^*_{1c} \\ & a^*_{22} & a^*_{23} & a^*_{24} & a^*_{2y} & a^*_{2c} \\ & & a^*_{33} & a^*_{34} & a^*_{3y} & a^*_{3c} \\ & & & a^*_{44} & a^*_{4y} & a^*_{4c} \\ & & & & a^*_{yy} & a^*_{yc} \end{array}$$

We then compute the following from (4):

$$(5)\qquad \begin{array}{ccccc|cc} & & & & & & \text{Check Column} \\ A_{11} & A_{12} & A_{13} & A_{14} & A_{1y} & A_{1c} & \\ B_{11} & B_{12} & B_{13} & B_{14} & B_{1y} & B_{1c} & \Sigma B_{1j}\,(j=1,2,3,4,y) \\ \hline & A_{22} & A_{23} & A_{24} & A_{2y} & A_{2c} & \Sigma A_{2j}\,(j=2,3,4,y) \\ & B_{22} & B_{23} & B_{24} & B_{2y} & B_{2c} & \Sigma B_{2j}\,(j=2,3,4,y) \\ \hline & & A_{33} & A_{34} & A_{3y} & A_{3c} & \Sigma A_{3j}\,(j=3,4,y) \\ & & B_{33} & B_{34} & B_{3y} & B_{3c} & \Sigma B_{3j}\,(j=3,4,y) \\ \hline & & & A_{44} & A_{4y} & A_{4c} & \Sigma A_{4j}\,(j=4,y) \\ & & & B_{44} & B_{4y} & B_{4c} & \Sigma B_{4j}\,(j=4,y) \\ \hline & & & & A_{yy} & A_{yc} & A_{yy} \end{array}$$

TABLE I

Source	d.f.	S.S.	M.S.	F
Regression	4	$a_{yy}-10^{-2p}A_{yy}$	$\dfrac{a_{yy}-10^{-2p}A_{yy}}{4}$	M.S. (Regression) / M.S. (Residual)
Residual	n−5	$10^{-2p}A_{yy}$	$\dfrac{10^{-2p}A_{yy}}{n-5}$	
Total	n−1	a_{yy}		

where

$A_{1j}=a^*_{1j}$

$B_{1j}=A_{1j}/A_{11}$

$A_{2j}=a^*_{2j}-A_{12}B_{1j}$

$B_{2j}=A_{2j}/A_{22}$

$A_{3j}=a^*_{3j}-A_{13}B_{1j}-A_{23}B_{2j}$

$B_{3j}=A_{3j}/A_{33}$

$A_{4j}=a^*_{4j}-A_{14}B_{1j}-A_{24}B_{2j}-A_{34}B_{3j}$

$B_{4j}=A_{4j}/A_{44}$

$A_{yj}=a^*_{yj}-A_{1y}B_{1j}-A_{2y}B_{2j}-A_{3y}B_{3j}\ -A_{4y}B_{4j}$

After completing the calculations for each row of (5), we compute the term indicated in the check column and compare it with the A_{ic} or B_{ic} term associated with it. If these do not agree within one of the last two decimal places, a mistake has been made, and the last row should be checked before proceeding.

Tests of Significance

We now have all the values needed to compute R^2 and the entries for the analysis of variance table. R^2 is computed by

$$(6)\qquad R^2 = \frac{a_{yy}-10^{-2p}A_{yy}}{a_{yy}}$$

The analysis of variance table for regression due to fitting all the x_i's is set up as in Table I. If one wishes to test the significance of the additional reduction due to any regression variable over that obtained for previous ones, the analysis of variance shown in Table II can be set up.

In fact, if the research worker is only interested in fitting those independent variables that add a significant reduction to the regression sum of squares, this can be accomplished by modifying the above. First we compute (4) in the usual way; then the first two rows of (5).

TABLE II

Source	d.f.	S.S.	M.S.	F
Regression due to x_1	1	$10^{-2p}A_{1y}B_{1y}=A$	A	A / M.S. (Residual)
Additional reduction due to x_2 over x_1	1	$10^{-2p}A_{2y}B_{2y}=B$	B	B / M.S. (Residual)
Additional reduction due to x_3 over x_1 and x_2	1	$10^{-2p}A_{3y}B_{3y}=C$	C	C / M.S. (Residual)
Additional reduction due to x_4 over x_1, x_2 and x_3	1	$10^{-2p}A_{4y}B_{4y}=D$	D	D / M.S. (Residual)
Residual	n−5	$10^{-2p}A_{yy}$	$\dfrac{10^{-2p}A_{yy}}{n-5}$	
Total	n−1	a_{yy}		

Now we compute an F-ratio where

$$F = \frac{10^{-2p}A_{1y}B_{1y}}{\frac{-10(a_{yy}-{}^{-2p}A_{1y}B_{1y})}{n-2}} \tag{7}$$

If this F-ratio which has one and (n−2) degrees of freedom is significant, we proceed to the next two rows of (5); if it is not significant, we delete the first row of (4), which has the effect of throwing out x_1. If at any time the residual sum of squares is reduced enough that $10^{-2p}A_{1y}B_{1y}$ would be significant, the variable x_1 can be brought back into (4) in another position. If this does not happen, it is left out of the analysis. To see if the added reduction due to x_3 is significant we compute

$$F = \frac{10^{-2p}A_{3y}B_{3y}}{\frac{(a_{yy}-10^{2p}\sum_{i=1}^{3}A_{iy}B_{iy})}{n-4}} \tag{8}$$

We would then proceed as above, if this F-ratio with one and (n−4) degrees of freedom were significant.

If the above F-tests are not significant, or if the analysis of Table I is not significant, the research worker will be able to stop here because the regression coefficients and their variances will not be required. If, however, the F-test in Table I was significant and the regression coefficients are required, we would proceed as follows.

First a set of modified regression coefficients, b^*_i, are calculated by

$$\begin{aligned} b^*_4 &= B_{4y} \\ b^*_3 &= B_{3y} - b^*_4B_{34} \\ b^*_2 &= B_{2y} - b^*_3B_{23} - b^*_4B_{24} \\ b^*_1 &= B_{1y} - b^*_2B_{12} - b^*_3B_{13} - b^*_4B_{14}. \end{aligned} \tag{9}$$

The required set of regression coefficients, b_i, are then computed from (9) and (2) using

$$\begin{aligned} b_1 &= 10^{(q_1-p)}b^*_1 \\ b_2 &= 10^{(q_2-p)}b^*_2 \\ b_3 &= 10^{(q_3-p)}b^*_3 \\ b_4 &= 10^{(q_4-p)}b^*_4. \end{aligned} \tag{10}$$

At this stage, we can make a further check on our calculations since $\sum_{i=1}^{4} b_ia_{iy} = a_{yy} - 10^{-2p}A_{yy}$, which is the regression sum of squares in Table I.

To obtain the variances of the b_i's we must obtain the diagonal terms of the inverse of the first four rows and columns of (1). These terms are obtained by first computing the inverse of (4) by the backward solution of the Abbreviated Doolittle Method.

Backward Solution of the Abbreviated Doolittle Method

The inverse of (4) is computed and recorded in the following manner:

$$\begin{array}{cccc} c^*_{11} & c^*_{12} & c^*_{13} & c^*_{14} \\ & c^*_{22} & c^*_{23} & c^*_{24} \\ & & c^*_{33} & c^*_{34} \\ & & & c^*_{44} \end{array} \tag{11}$$

where

$$\begin{aligned} c^*_{44} &= 1/A_{44} \\ c^*_{34} &= -c^*_{44}B_{34} \\ c^*_{24} &= -c^*_{34}B_{23} - c^*_{44}B_{24} \\ c^*_{14} &= -c^*_{24}B_{12} - c^*_{34}B_{13} - c^*_{44}B_{14}. \end{aligned}$$

At this point another check is applied by computing $\sum_{i=1}^{4} a^*_{i4}c^*_{i4}$, which should be approximately one.

$$\begin{aligned} c^*_{33} &= 1/A_{33} - c^*_{34}B_{34} \\ c^*_{23} &= -c^*_{33}B_{23} - c^*_{34}B_{24} \\ c^*_{13} &= -c^*_{23}B_{12} - c^*_{33}B_{13} - c^*_{34}B_{14} \end{aligned}$$

One should now check to see that $\sum_{i=1}^{4} a^*_{i3}c^*_{i3}$ is approximately one.

$$\begin{aligned} c^*_{22} &= 1/A_{22} - c^*_{23}B_{23} - c^*_{24}B_{24} \\ c^*_{12} &= -c^*_{22}B_{12} - c^*_{23}B_{13} - c^*_{24}B_{14} \end{aligned}$$

Again check to see if $\sum_{i=1}^{4} a^*_{i2}c^*_{i2}$ is approximately one.

$$c^*_{11} = 1/A_{11} - c^*_{12}B_{12} - c^*_{13}B_{13} - c^*_{14}B_{14}$$

The last check is that $\sum_{i=1}^{4} a^*_{i1}c^*_{i1}$ should be approximately one.

In the above it is obvious because of symmetry that $a^*_{ij} = a^*_{ji}$ and $c^*_{ij} = c^*_{ji}$.

Now to obtain the variances of the regression coefficients, we require c_{11}, c_{22}, c_{33}, c_{44}. These are calculated by

$$c_{ii} = 10^{2q_i}c^*_{ii}. \tag{12}$$

The variance of b_i is then obtained by multiplying c_{ii} times the residual mean square.

Illustrative Example

The above procedure will be applied to a regression problem which has four independent variables, one dependent variable, and 57 observations. The numbers in parentheses and the table numbers refer back to the algebraic section so that one can see how each step was obtained.

MATRIX 5

						Check Column
0.8238930	−1.1309660	−0.8636493	0.0607200	−0.5265790	−1.6365813	
1	−1.3727098	−1.0482542	0.0736989	−0.6391352	−1.9864003	−1.9864003
	0.1496239	0.1919546	−0.0665490	0.0176668	0.2926963	0.2926963
	1	1.2829140	−0.4447752	0.1180747	1.9562135	1.9562135
		1.0623368	0.2476476	0.3365022	1.6464866	1.6464866
		1	0.2331159	0.3167566	1.5498725	1.5498725
			0.7772951	0.0143720	0.7916671	0.7916671
			1	0.0184898	1.0184898	1.0184898
				0.0643388	0.0643388	0.0643388

TABLE I

Source	d.f.	S.S.	M.S.	F
Regression	4	4454.96	1113.74	90.04
Residual	52	643.39	12.37	
Total	56	5098.35		

After obtaining the following sums of squares and sums of products

$$(1)\qquad \begin{matrix} 8{,}238.93 & -11{,}309.66 & -86{,}364.93 & 60.72 & -5{,}265.79 \\ & 17{,}021.12 & 137{,}749.45 & -149.90 & 7{,}405.07 \\ & & 2{,}213{,}922.00 & 986.21 & 91{,}115.58 \\ & & & 86.91 & 46.15 \\ & & & & 5{,}098.35, \end{matrix}$$

we construct the diagonal matrix,

$$(2)\qquad \begin{bmatrix} 10^{-2} & 0 & 0 & 0 & 0 \\ 0 & 10^{-2} & 0 & 0 & 0 \\ 0 & 0 & 10^{-3} & 0 & 0 \\ 0 & 0 & 0 & 10^{-1} & 0 \\ 0 & 0 & 0 & 0 & 10^{-2} \end{bmatrix}$$

Pre- and post-multiplying (1) by (2) we obtain

$$(3)\qquad \begin{matrix} 0.8238030 & -1.1309660 & -0.8636493 & 0.0607200 & -0.5265790 \\ & 1.7021120 & 1.3774945 & -0.1499000 & 0.7405070 \\ & & 2.2139220 & 0.0986210 & 0.9111558 \\ & & & 0.8691000 & 0.0461500 \\ & & & & 0.5098350 \end{matrix}$$

It should be noted that there never is more than one digit before the decimal point in any element of (3), and that all elements have the same number of decimal places.

By adding the terms in each row (including those not shown because of symmetry) we obtain

$$(4)\qquad \begin{matrix} 0.8238930 & -1.1309660 & -0.8636493 & 0.0607200 & -0.5265790 & -1.6365813 \\ & 1.7021120 & 1.3774945 & -0.1499000 & 0.7405070 & 2.5392475 \\ & & 2.2139220 & 0.0986210 & 0.9111558 & 3.7375440 \\ & & & 0.8691000 & 0.0461500 & 0.9246910 \\ & & & & 0.5098350 & 1.6810688 \end{matrix}$$

The forward solution of the Abbreviated Doolittle Method is shown in matrix (5).

We then find

$$(6)\qquad R^2 = \frac{5098.35 - 10^4(0.0643388)}{5098.35} = 0.8738,$$

the sum of squares due to regression is $5098.35 - 10^4(0.0643388) = 4454.96$, and the residual sum of squares is $10^4(0.0643388) = 643.39$. These are the entries for Table I as shown.

TABLE II

Source	d.f.	S.S.	M.S.	F
Regression due to x_1	1	3365.55	3365.55	272.07
Additional reduction due to x_2 over x_1	1	20.86	20.86	1.69
Additional reduction due to x_3 over x_1, x_2	1	1065.89	1065.89	86.17
Additional reduction due to x_4 over x_1, x_2, x_3	1	2.66	2.66	—
Residual	52	643.39	12.37	
Total	56	5098.35		

We can also compute the entries for Table II if they are desired.

Regression due to $x_1 = 10^4(-0.5265790)(-0.6391352) = 3365.55$

Additional reduction due to x_2 over $x_1 = 10^4(0.0176668)(0.1180747) = 20.86$

Additional reduction due to x_3 over x_1, $x_2 = 10^4(0.3365022)(0.3167566) = 1065.89$

Additional reduction due to x_4 over x_1, x_2, $x_3 = 10^4(0.0143720)(0.0184898) = 2.66$

Since the regression in Table II due to all the variables was significant as seen in Table I, we will find the regression coefficients.

$$(9)\qquad \begin{aligned} b^*_4 &= 0.0184898 \\ b^*_3 &= 0.3167566 - (0.0184898)(0.2331159) \\ &= 0.3124463 \\ b^*_2 &= 0.1180747 - (0.3124463)(1.2829140) \\ &\quad - (0.0184898)(-0.4447752) \\ &= -0.2745432 \\ b^*_1 &= -0.6391352 - (-0.2745432)(-1.3727098) \\ &\quad - (0.3124463)(-1.0482542) \\ &\quad - (0.0184898)(0.0736989) \\ &= -0.6898429 \end{aligned}$$

and

$$(10)\qquad \begin{aligned} b_1 &= 10^0(-0.6898429) = -0.6898429 \\ b_2 &= 10^0(-0.2745432) = -0.2745432 \\ b_3 &= 10^{-1}(0.3124463) = 0.0312446 \\ b_4 &= 10^1(0.0184898) = 0.1848980 \end{aligned}$$

We can now compute $\sum_{i=1}^{4} b_i a_{iy}$ and we see that it is 4454.96, which checks perfectly.

Now, in order to find the variance of b_i, we compute the matrix (11) whose elements are c^*_{ij}

$$(11)\qquad \begin{matrix} 14.9216670 & 10.7079820 & -0.8818266 & 0.9044400 \\ & 8.9445450 & -1.4307173 & 0.9569633 \\ & & 1.0112340 & -0.2999066 \\ & & & 1.2865127. \end{matrix}$$

Adjustment of the elements of (11) yields.

$$(12)\qquad \begin{aligned} c_{11} &= 0.0014921667 \\ c_{22} &= 0.0008944545 \\ c_{33} &= 0.0000010112 \\ c_{44} &= 0.0128651270, \end{aligned}$$

so that all we need to do now is multiply the c_{ii}'s by the residual mean square to obtain the variances of the regression coefficients.

Discussion

We have made two simple innovations in the use of the Modified Doolittle Method for multiple regression. The a_{iy} term was introduced in the forward solution and adjustments of decimal places for uniformity were introduced. While these modifications are simple, they are extremely useful and convenient in the numerical computations for problems in multiple regression.

References

1. Anderson, R. L. and Bancroft, T. A.; *Statistical Theory in Research;* First Edition, McGraw-Hill Book Company, Inc. 1952; Chap. 15
2. Duncan, D. B. and Kenney, J. F.; *On the Solution of Normal Equations and Related Topics;* Edwards Brothers, Inc., Ann Arbor, Michigan, 1946
3. Peach, Paul; *The Abbreviated Doolittle Method for Matrix Inversion;* Institute of Statistics, Mimeo. Series 28, Raleigh, N. C.

The Principles of Experimental Design*

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The Nature and Purposes of Experimentation

A Definition

When I read the announcement of this colloquium that "the emphasis in these lectures will be directed toward the philosophy and practical use of statistics" I started to think about the philosophy of experimental design. I began to wonder (a) what an experiment is, and (b) why we do them. I suppose our reaction to the question of what an experiment is depends on our occupation: a physicist would have different ideas from those of a chemist or a microbiologist or an engineer; an astronomer or meteorologist or geologist, on the other hand, might be lacking in ideas, since, at least at first sight, they deal with observations rather than experiments.

Webster's dictionary defines an experiment as "a trial or special observation made to confirm or disprove something doubtful, especially one under conditions determined by the experimenter; an act or operation undertaken in order to discover some unknown principle or effect or to test, establish, or illustrate some suggested or known truth."

Those of us who were educated in some pure science, such as physics, with the emphasis on the scientific and not on the engineering aspects, learnt to think of experiments as classical operations in the history of our science; Cavendish weighed the earth, Joule measured the mechanical equivalent of heat, Millikan measured the charge on the electron, and so on. It may be worth-while to try to compare the kind of experiments we (that is, you and I) perform in our daily employment.

Comparative and Absolute Experiments

Anscombe[1] has emphasized the distinction between

(a) absolute experiments
(b) comparative experiments.

In general the distinction seems to hold up to scrutiny. Our physicist measuring the weight of the earth is making an absolute experiment, our chemical engineer comparing electric power consumption for two different positions of the electrodes is making a comparative experiment. On the other hand, a lot of atomic weight determinations might appear at first sight to be absolute, but in point of fact consist of the determination of the ratio of two atomic weights, so are comparative in that sense. However, a great deal of applied science appears to fall into the category of the comparative experiment: we are comparing the yield of a chemical reaction under different conditions, the tensile strength of alternative alloys, the lives of components manufactured somewhat differently.

Another useful distinction between various classes of experiments that might be made is whether the experiment is designed for the purpose of

(a) adding to our knowledge of the universe
(b) enabling us to make a satisfactory administrative decision.

The former is presumably pure science, the latter applied science. Since the methods, techniques, processes tend to be similar in the two forms of activity, we tend to assume that the activities are identical without often stopping to consider whether this is really so.

Sometimes the distinction is clear. The physicist weighing the earth is not concerned with any administrative decision; in the light of present technology it is difficult to conceive of any decision for which his result would be relevant. Note the qualification "in the light of present technology." It may well be that to some later generation in the dim future this is an important practical point, determining the choice between two alternative methods of moving the earth's orbit nearer the sun. The chemical engineer is not usually contributing to fundamental knowledge when he shifts his electrodes, he is concerned solely with finding the position which gives the maximum profit (a noble pursuit, I may remark). There are, however, wide fields in which the situation is somewhat ambiguous. For example, the work being done on the effects of radiation on mammals might be considered as contributions to pure knowledge and also as useful for making certain administrative decisions. In fact, I suspect that a lot of the applied science that gets done is in fact a compromise: the researcher is careful to satisfy the objectives of the organization which is paying him but at the same time he is hoping that he can work in tests of his own pet pure scientific theories.

On the whole, it appears as if there are the two correspondences:

(a) comparative experiments leading to administrative decisions;
(b) absolute experiments leading to pure scientific knowledge.

However, as indicated above, overlap occurs and clear distinctions cannot be made.

The Techniques of the Design of Experiments

The modern science of the design of experiments grew up largely under the tutelage of Fisher and Yates at Rothamsted Experiment Station very much in the field of comparative experiments, in agronomy. It is perhaps for this reason that we have been able to take over their methods into comparative experiments in so much of applied science.

In my thinking at the present time I have not found from the point of view of the consulting statistician any very substantial differences between the various categories of experiments. There are some, of course. For example, the device of confounding was developed in the science of agronomy to minimize the effects of field heterogeneity. A field experiment in agronomy may be designed to compare various fertilizer treatments, for example. It is, usually, strictly a comparative experiment. We are solely interested in the relative yields obtained by the

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various treatments; any interest there may be in the absolute values is very secondary. It is known by experience in agronomy that the absolute value varies from plot to plot and more so from group of plots to group of plots. To minimize the effects of this variability the device of confounding was introduced. In an agricultural experiment, therefore, the existence of appreciable variation between blocks is of mild interest, as for example, showing that splitting the experiment up into blocks was a good thing, but not of direct importance to the main objective. In an absolute experiment, on the other hand, variation in the background is clear indication that something is wrong. For example, if we are measuring the ratio of the electromagnetic to the electrostatic unit of electricity, and we make a number of determinations, dismantle the apparatus, clean it and reassemble it and make another group of determinations, then each group is analogous to a block in the agricultural experiment. But whereas in the agricultural experiment it would be irrelevant if there was variation from block to block, in this experiment it would be critical, since it would mean that there were uncontrolled factors present giving rise to sources of error which should be absent.

Objectives in the Design of Experimentation

It is clear that we, here, are concerned today with applied science, comparative experiments leading to administrative decisions, and our remarks will now be concerned only directly with this type of experiment.

In designing an experiment we may have one of two objectives:

(a) to test whether an effect exists

(b) to measure the magnitude of the effect.

If our objective is testing, we will have two main criteria:

(a) to minimize the probability of rejecting the null hypothesis when it is true;

(b) to minimize the probability of accepting the null hypothesis when it is false.

These are the so-called errors of the first and second kinds. Just what weight we give to the relative values of the two probabilities depends on the relative consequences of the two kinds of errors. The consequences may be measured in dollars or in social values, or in a mixture of the two.

For example, if we are testing whether adding a certain ingredient to an alloy increases the tensile strength, the consequences of failing to detect that it has an effect will increase with the magnitude of the effect. It is fortunate that in a complementary way the probability of our finding the effect also increases with its magnitude. Here, the consequences of rejecting the null hypothesis when it is actually true are usually not very serious, since if we act accordingly and start to use the ingredient, it may become apparent that no improvement in tensile strength has resulted. This is not always the case, of course. In medical research it is quite easy, and I am sure has happened often, for some new treatment to be proclaimed as superior to the old and be widely adopted and be considered as superior since an improvement is observed, whereas in fact there was no real difference between the treatments, and the improvement which was observed was due solely to a coincidental improvement in conditions of nutrition and general standard of living, or to a secular decline in the virulence of the disease.

Our criteria for a valid experiment includes

(a) an unbiased estimate of the effect;

(b) an unbiased estimate of the error in the estimate due to random variability.

It is my view that except perhaps for certain highly sophisticated or pathological circumstances these are the two prime requirements. A subsidiary requirement might be

(c) the error should be as small as possible.

Clearly, the smaller the error the smaller the chance of our committing errors of the first or second kinds.

We can achieve (a) and (b) by the device of randomization.

Randomization

The idea of randomization is absolutely fundamental to the design of valid experiments. I think this is one of Fisher's greatest contributions to the subject.

Randomization plays two parts. The first, the avoidance of bias, is relatively obvious, though it is remarkable how many apparently intelligent people try to resist it. Clearly, if all replications of one treatment occur on one set of plots and all replications of the other treatment on another set, then any apparent difference between the treatment means could be equally well ascribable to differences between the sets of plots. Stated in the extreme form, no one would dispute the necessity for randomization.

In a field experiment, the errors in adjacent plots will tend to be correlated, usually positively. If randomization is not employed this has a peculiarly vicious effect:

(a) If the correlation is positive, the treatment means have larger variances than if we had independent observations. In addition, the estimates of the variances of the treatment means which we obtain are too small. The probability of our making errors of the first kind, that is, rejecting the null hypothesis that an effect does not exist, will therefore be much higher than we think.

(b) Negative correlation is rare. Its effects are the inverses of those of positive correlation, and though less unpleasant are still not desirable.

The great beauty of randomization is that it destroys the correlation system, whatever it may be, and gives us uncorrelated and independent errors, and our standard significance tests are then valid. This wonderful fact cannot be too heavily emphasized.

Some of the reasons some experimenters advance in trying to resist the introduction of randomization into their work are that

(a) there is no reason to suppose that there will be a bias in this instance;

(b) it means much more work;

(c) things might get mixed up.

As regards (a), one might rephrase it to say "there is no reason to suppose that there will *not* be a bias in this instance." As regards (b), one might ask, "more than what?" for that a valid experiment takes more work than an invalid experiment is irrelevant to a man who is wanting to make valid inferences. As regards (c), one feels sympathy, but if an experimenter isn't willing to do a decent job why doesn't he choose some other easier way of earning a living? There is no need to argue about this: if we do not have randomization we do not have a valid experiment.

Summary

To summarize our discussion so far, the main contributions the statis-

TABLE I

Casting	Order of Segment 1	2	3	4	5	6	7
A	77.0	79.2	74.3	75.7	75.0	74.2	75.5
B	78.9	76.9	81.3	76.8	76.6	75.9	76.7
C	78.6	77.6	77.3	77.3	75.8	75.6	75.0
D	78.1	82.0	78.9	79.4	78.2	78.0	78.4
E	79.1	77.9	79.3	77.8	78.7	76.2	77.0
F	81.2	79.2	77.4	78.4	78.7	77.3	76.7
G	80.7	82.1	78.8	81.1	77.8	78.4	77.1

tician makes to the design of an experiment are (or should be)

(a) a careful examination of the objectives of the experiment,

(b) an insistence on the necessity for randomization,

(c) the mobilization of the technical tricks of experimental design.

In addition, with a complex experiment almost certainly a complex analysis of variance will be necessary to interpret the results, and this is going to call for skill and experience in this art. Experience will also be required in deciding how far the assumptions being made in the analysis are being satisfied: for example, we generally assume the errors to be

TABLE II
Random Allocation to Seven Treatments

Casting	Order of Segment						
A	1	2	7	5	3	6	2
B	3	7	1	5	5	4	3
C	6	2	3	5	4	3	1
D	4	2	4	1	4	1	2
E	1	5	2	6	7	2	5
F	4	7	4	7	2	4	1
G	5	6	1	6	4	5	2

(a) normally distributed,

(b) statistically independent,

(c) constant over the region of experimentation, i.e., homoscedastic.

It is rarely that any one experiment provides complete internal checks on these assumptions; we usually have to draw on our previous experience with similar systems. The independence, of course, we usually achieve by randomization. The normality and homoscedasticity can often be achieved by using some transformation of the dependent variable (see, for example, Bartlett[3] for a convenient summary).

We finally turn to consider that bag of tricks which is often what is meant by the phrase "the design of experiments." These tricks are systems of organizing the execution of the experiment, in space and time, to minimize the effects of heterogeneity of the experimental background. A series of examples will make clear what is meant.

Some Single Factor Experiments

An Example

The data we are going to use as an example refers to a physical property of a material that is cast in molds in an upright position. Seven castings were made and the castings then cut into seven segments. No treatment was applied to the segments, as this experiment was intended to ascertain sources of error, but we will suppose the objective was to compare seven separate treatments. There could be different aging times of the material before tests, or aging at different temperatures, for example. (Table I)

Completely Randomized Design

Let us suppose that the seven (hypothetical) treatments were allocated at random to the 49 "plots" (experimental units). By taking a sequence of random numbers from one to seven and allocating them serially from left to right starting at the top we obtained the arrangement in Table II. There are, in this instance eight observations with treatment 1, nine with treatment 2, five with treatment 3, and so on. Of course, in general, different results will be obtained each time we randomize. The actual arrangement of the observations corresponding to the allocation of Table II is given in Table III. For example, for treatment 1 the observations are the first with casting A (77.0), the third with casting B (81.3), the seventh with casting C (75.0), etc.

The appropriate analysis of variance of these data is a one-way analysis with unequal column sizes and is in Table IV.

TABLE III

Treatment	Observation									n_i
1	77.0	81.3	75.0	79.4	78.0	79.1	76.7	78.8		8
2	79.2	75.5	77.6	82.0	78.4	79.3	76.2	78.7	77.1	9
3	75.0	78.9	76.7	77.3	75.6					5
4	75.9	75.8	78.1	78.9	78.2	81.2	77.4	77.3	77.8	9
5	75.7	76.8	76.6	77.3	77.9	77.0	80.7	78.4		8
6	74.2	78.6	77.8	82.1	81.1					5
7	74.3	76.9	78.7	79.2	78.4					5

TABLE IV

Source of Variance	D.F.	SS	M. Sq.
Between Treatments	6	14.10947	2.35158
Within Treatments	42	156.42853	3.72448
Total	48	170.538	(3.55287)

In this case the treatments were hypothetical, so we will get a better estimate of the residual variance by pooling the sums of squares and degrees of freedom to get $s^2=3.55287$ as given in parentheses in the last line.

One awkwardness with an unbalanced design like the one in Table II is that the variances of all comparisons are not the same. For example, the variance of the difference between the mean for treatment 1 and treatment 2 is $s^2(\frac{1}{8} + \frac{1}{9}) = 0.236\ s^2$, whereas the variance of the comparison of treatment 6 with treatment 7 is $s^2(\frac{1}{5} + \frac{1}{5}) = 0.400\ s^2$. However, to get an average picture of the efficiency of this design we can calculate the variances of all $\binom{7}{2} = 21$ comparisons and take the average, obtaining $0.30629\ s^2 = 0.30629 \times 3.5529 = 1.088$. This result is tabulated in the first line of Table XI.

Randomized Balanced Design

As a modification of this design let us again take the seven treatments at random, but impose the restriction that there shall be only seven replicates of each. This we can do by proceeding as before, but when we have seven replicates of any treatment, deleting that treatment from the group from which we are picking treatments at random. The result of one performance of such a randomization procedure is in Table V.

TABLE V—Random Allocation to Seven Treatments with Restriction to Equal Numbers of Replicates

Casting	Order of Segment						
A	5	3	7	4	2	3	6
B	7	6	1	3	2	2	6
C	4	4	6	2	6	7	6
D	2	4	3	3	4	1	1
E	5	4	5	3	5	3	6
F	5	4	5	2	2	7	7
G	7	7	5	1	1	1	1

The appropriate analysis of variance for Table V is a one-way analysis with equal sized columns (Table VI).

TABLE VI

Source of Variance	D.F.	SS	M. Sq.
Between Treatments	6	36.935	6.1558
Within Treatments	42	133.603	3.1810
Total	48	170.538	(3.5529)

Here again we get a better idea of the residual variance by pooling, since the treatments were hypothetical. The variance of the difference between any two treatment means will be $s^2(1/7 + 1/7) = 1.0151$ (see line 2 in Table XI). It will be noted that this is a slight improvement over the average obtained for the preceding design 1.088. This illustrates the general rule that a balanced design is always more efficient than an unbalanced. In addition, the analysis of a balanced design is simpler. In fact, in complex situations unbalanced designs may be unanalyzable for all practical purposes.

Randomized Blocks

Now suppose we allocate the seven treatments at random to the seven segments within each casting. One such arrangement is in Table VII.

TABLE VII—Random Allocation to Seven Treatments with Restriction That All Seven Treatments Occur with Each Casting

Casting	Order of Segment						
A	2	5	7	3	6	1	4
B	4	2	1	7	3	6	5
C	4	6	5	1	2	3	7
D	1	7	3	2	4	5	6
E	4	5	1	7	6	2	3
F	7	3	4	1	6	2	5
G	7	3	5	6	4	2	1

We then can arrange the data in a two-way table with rows (= to castings) and columns (= to treatments). The resulting analysis of variance is in Table VIII.

TABLE VIII

Source of Variance	D.F.	SS	M. Sq.
Between Castings	6	66.353	11.0588
Between Treatments	6	3.558	0.5930
Remainder	36	100.627	2.7952
Total	48	170.538	

Here again we get a better estimate of the residual by pooling the remainder with the treatment sum of squares. The new residual mean square is 2.4806 with 42 degrees of freedom. The variance of the difference between the means of any two treatments will be $s^2(1/7 + 1/7) = 0.7087$ (line 3 in Table XI). This is a substantial decrease from the previous design.

TABLE IX—Random Allocation to Seven Treatments with Restriction That All Seven Treatments Occur with Each Casting and with Each Segment

Casting	Order of Segment						
A	2	7	3	4	1	6	5
B	3	1	4	5	2	7	6
C	6	4	7	1	5	3	2
D	4	2	5	6	3	1	7
E	5	3	6	7	4	2	1
F	1	6	2	3	7	5	4
G	7	5	1	2	6	4	3

Latin Square Arrangement

In the case where the number of treatments equals the number of blocks, it is further possible to balance the arrangement with respect to a third categorization. Consider the arrangement of treatments in Table IX. It will be noted that each row, corresponding to castings, contains all seven treatments. In addition, each column, corresponding to order of segment, contains all seven castings. This balanced relationship is symmetrical with all three categorizations—rows, columns, and treatments. Thus each row and column occurs with each treatment. Such an arrangement is known as a Latin Square. The analysis of variance is in Table X.

TABLE X

Source of Variance	D.F.	SS	M. Sq.
Rows (Castings)	6	66.353	11.0588
Columns (Order of Segments)	6	51.353	8.5588
Treatments	6	4.775	0.7958
Remainder	30	48.057	1.6019
Total	48	170.538	

As the treatments were here "dummy" we will get a better estimate of the residual mean square by pooling, obtaining $s^2 = 1.4676$. The variance of the difference between any two treatment means will be $s^2(1/7 + 1/7) = 0.4193$ (see line 4 in Table XI).

Comparison of the Foregoing Designs

We can assemble the results for these various experimental designs in Table XI. The final column gives the variances relative to the best available here, the Latin Square. The extra number of replicates needed to bring down the variance to the figure for the Latin Square is directly proportional to the inverse of this figure. For example, with the first arrangement we would need in this instance $1/0.385 = 2.60$ times as many observations to get the same accuracy. The actual efficiencies are of course a function of the heterogeneity of the experimental background in the particular example in question; i.e., the effects due to differences between castings and order of segments. If the variation has been completely at random over all castings and there had been no order of segment effect, then the Latin Square would have led to no reduction in the error variance, and in point of fact we would have lost slightly as the degrees of freedom for the error variance would have been reduced by using the Latin Square. For a 7×7 square this loss would not have been noticeable, but if we had been using a 4×4 square the reduction of degrees of freedom from 12 to 6 would have appreciably affected the power of the experiment.

Graeco-Latin and Orthogonal Squares

It would have been possible to introduce a further restriction into the experiment which we have been discussing. Suppose rather than measure all 49 experimental units on one measuring machine we would prefer to measure them on seven machines to speed up the experiment. Further suppose that there are, or we suspect there are, systematic differences in the zeros of the measuring machines, so that they tend to read low or high by constant amounts. If we allocate the experimental units to the machines denoted by the letters A to G

TABLE XI

Type of Design	Variance of Comparison of Two Means	Efficiency Relative to Latin Square
Random Allocation	1.088	38.5
Restriction that treatments occur equal number of times	1.015	41.3
Restriction that treatments occur equal number of times (once) in each block ("Randomized Blocks")	0.7087	59.2
Further restriction that treatments occur equal number of times (once) in each column ("Latin Square")	0.4193	100

TABLE XII

A1a	B2b	C3c	D4d	E5e	F6f	G7g
D7f	E1g	F2a	G3b	A4c	B5d	C6e
G6d	A7e	B1f	C2g	D3a	E4b	F5c
C5b	D6c	E7d	F1e	G2f	A3g	B4a
F4g	G5a	A6b	B7c	C1d	D2e	E3f
B3e	C4f	D5g	E6a	F7b	G1c	A2d
E2c	F3d	G4e	A5f	B6g	C7a	D1b

as in Table XII, we may note that each level of the new categorization occurs once and once only with each level of the rows, columns, and treatments, and conversely. Arrangements of this type are known as Graeco-Latin Squares, because instead of numbers and Latin letters in the cells as in Table XII we can use Greek and Latin letters.

In the present case where the square is of side seven it is possible to introduce a still further restriction. Suppose that the material is affected by the humidity of the atmosphere, and we can only make one measurement per machine per day. In the arrangement in Table XII the small letters represent the different days on which the measurements are taken, and it will be noted that each day occurs once and once only with each row (casting), column (segment), number (treatment), and Latin letter (measuring machine).

In general, Bose[3] and Stevens[4] showed that where n, the side of the square, is a prime number or power of a prime, it is possible to introduce (n−1) categorizations (alphabets) in addition to the rows and columns. The only value of n between two and nine which is not of this type is six, and for squares of this side only one alphabet may be introduced. An early industrial example of a 7 × 7 Graeco-Latin square was reported by H. M. Davis[5]. He used it to compare seven gasolines for miles per gallon with seven drivers on seven days of the week and seven times of day.

Balanced Incomplete Blocks and Youden Squares

The applicability of the randomized block idea to the experiment discussed earlier depended on the number of segments from each casting being equal to the number of treatments under test, and the Latin Square arrangement could only be applied because the number of castings available also equalled the number of treatments.

TABLE XIII—Random Allocation of Seven Treatments to First Four Segments in Each Casting, Balanced Incomplete Block Design

Casting	Order of Segment			
A	3	5	6	7
B	4	6	7	1
C	5	7	1	2
D	6	1	2	3
E	7	2	3	4
F	1	3	4	5
G	2	4	5	6

To consider the first type of design, randomized blocks, for certain combinations of numbers of treatments and blocks and size of blocks it is possible to have the block size smaller than the number of treatments and still have a design for which the effects of differences between blocks can be readily eliminated by comparatively simple computation. These designs are due to Yates[6]. The fundamental property is that the ith treatment should occur together with the jth treatment in the same block a certain number λ of times.

Suppose that there were only four segments per casting but that we still wished to compare seven treatments. The arrangement in Table XIII has the necessary property referred to above. Thus treatment 1 occurs with treatment 2 in castings three and four, or twice, and all pairs of treatments occur together in the same block this number of times.

Actually, the design of Table XIII has a further important property, namely that each column contains each treatment once and only once. It is thus a member of the subclass of balanced incomplete blocks known as Youden Squares[7].

Frazier[8] has published an example of a 13 × 4 Youden Square used for comparing 13 oils for their oil consumption and gasoline consumption in 13 automobiles in normal use over four periods.

Summary

The foregoing example has been intended to illustrate the device of

(a) randomized blocks
(b) Latin squares
(c) Graeco-Latin Squares
(d) balanced incomplete blocks
(e) Youden Squares.

The main idea was to try to show how by modifying the organization of an experiment we can increase its accuracy very substantially for the same amount of work. Part of the increase in accuracy came from the imposition of balance. The greater part of the increase, however, came from removing from the error sum of squares those parts due to two extraneous sources of variation, namely rows (= castings) and columns (= order of segment).

It is perhaps not sufficiently realized that in principle we could have removed these extraneous sources of variation even without the use of a balanced design, by setting up an enormous multiple regression equation with parameters corresponding to each row, etc. The variable corresponding to a given row takes the value 1 if that particular observation happens to fall in that row but is otherwise zero. The brutal fact remains, however, that to solve the matrix for fitting such an equation would involve a preposterous amount of work, and only in very exceptional circumstances would we be justified in undertaking it. Therefore the main function of good design is just to make complicated analysis easy and practicable.

Some Distinctions in the Analysis of Variance

Model I and Model II

In almost all complex experiments we use the analysis of variance to interpret the results. It is therefore important to remember that there are two main types of analysis of variance[9, 10]:

(a) that in which we are seeking to estimate Model I or fixed effects. These are a set of constants, for example, the yields at a set of specified temperatures;

(b) that in which we are seeking to estimate Model II or random effects. Here we have no direct interest in the particular set of row and column means that happened to turn up in our particular experiment; our interest in them is solely as a random sample from a supposedly normal distribution. We use the particular observations we obtained to estimate the parameters of that distribution, namely the mean and variance;

(c) a third type, namely the Mixed Model, in which some effects are random and some are fixed.

Latin Squares and Models I and II

The original usage of the Latin square was as a device for reducing the effects of soil heterogeneity in agronomy. The rows and columns of the squares corresponded to rows and columns on the field, and they were regarded as random samples from some hypothetical population of rows and columns. Any deviations from an additive model, i.e., row × column, row × treatment, and column × treatment interactions will also be random variances and belong

in the error mean square. It is therefore not necessary to assume that these interactions do not exist; their presence is somewhat undesirable, since they will increase the experimental error, but if they do exist, we can accept them as being a genuine source of error.

In recent technological experimentation there has grown up a tendency to use Latin Squares as fractional replications of the full factorial experiment. The 5×5 Latin square, for example, requires 25 plots, and is a one-fifth replicate of the 5^3 full factorial experiment which requires 125 plots. The 5×5 Graeco-Latin Square is a 1/25 replicate of the 5^4 full factorial which requires 625 plots. In such a technological usage of a Latin Square the three categories—rows, columns, and treatments—may correspond to three variables like temperature, pressure, and flow rate; it will be noted that now they are all Model I variables, made up of sets of fixed constants. The interactions, instead of being random variables, will be sets of fixed constants, and the remainder mean square will estimate these in addition to the true error. We thus only have a satisfactory analysis if we can assume that these interactions are zero.

Thus in a technological experiment using a Latin Square where the rows and columns correspond to fixed effects we are actually in a different situation from the original agronomical use of the design, and are relying more heavily upon the non-existence of the interactions.

In the earlier example of the castings, the castings themselves, corresponding to the rows, are a random effect, but the order of segment, corresponding to columns, is a fixed effect. The castings are presumably a random sample of a population, but the order of segment effect can be represented by a set of fixed constants corresponding to each position.

Multi-Factor Experiments

Factorial Experiments

So far we have been discussing a single factor experiment. The seven hypothetical treatments were alternative treatments: they were alternative levels of a single factor. There is another whole series of classes of experimental designs which refer to testing several factors at various levels; this is the so-called factorial experiment. If we have one factor at two levels, one at three levels and one at four levels, for example, the experiment can be conveniently described by the symbol $2 \times 3 \times 4$; if we had three factors at four levels we could use the symbol $4 \times 4 \times 4$, or more compactly 4^3.

The full factorial experiment consists in running all combinations of all factors at all levels. The total number of runs required is then the same as given by the symbol for the experiment; for example, the 4^3 experiment requires 64 runs.

The factorial experiment has the great advantage of

(a) measuring main effects more accurately than the simple, one-variable-at-a-time experiment;

(b) measuring interactions between one factor and another. These interactions can be important and can only be detected by one form or other of a factorial experiment.

The factorial experiment has been developed in two directions:

(a) For experiments of the type π^n, i.e., n factors all at π levels, where π is a prime number, elegant schemes are available for splitting up the experiment into a number, m, of blocks, so that each block contains only π^{n-m} plots. Increased accuracy usually results. This device is known as *confounding*.

(b) For experiments of the type π^n, when n is rather large, schemes are available for replicating only a fraction of the π^n plots without losing information on the first order interactions. This device is known as *fractional replication*.

(c) The techniques of (a) and (b) can be extended to certain experiments of the type $\pi^{n_1}_1 (\pi^2)^{n_2}$. For example, for $\pi = 2$, $n_1 = 4$, $n_2 = 1$ we have the $2^4 \times 4^1$ experiment, or four factors at two levels and one factor at four levels.

(d) The device of confounding can be applied to fractionally replicated experiments.

I will not discuss these various developments at this time. Instead we will consider an experiment which illustrates a commonly occurring feature, namely a split-plot structure.

An Example

As an example of a factorial experiment I will discuss some data on a comparison of the yields obtained from different batches of the main ingredient of a medium for a fermentation for the production of an antibiotic. The prime objective was their comparison, but it was considered possible that the batch might interact with the strain of the mold used, in other words, that the relative merits of the batches might depend on the strain. It was also considered possible that the batch might interact with the concentration at which it was used. Finally it was considered possible that the batch might interact with the age of the fermentation. Thus, in order for our inferences about the relative merits of the four batches to have any substantial basis for generality, we needed to include in the experiment three other factors: concentration, strain, and age.

A difficult question is to decide the number of levels at which these four factors are to be tested. The number of batches was fixed. The number of strains was limited to those in use, namely four. Three concentrations were selected, as this

TABLE XIV

Batch	Strain	Age A_0			Age A_1		
		Concentration					
		1	2	3	1	2	3
1	A	43	50	62	40	69	70
	B	52	78	109	52	71	91
	C	51	73	89	78	100	110
	D	56	73	108	59	76	108
2	A	50	56	69	47	76	91
	B	68	80	125	64	72	99
	C	57	77	75	73	122	143
	D	71	85	111	77	106	127
3	A	50	60	85	55	79	102
	B	71	107	117	61	83	94
	C	57	84	93	71	106	106
	D	75	97	108	78	103	127
4	A	43	64	72	44	77	85
	B	67	80	106	69	75	116
	C	74	89	99	87	106	131
	D	63	121	77	76	107	125

is the minimum number of points that will allow possible curvature in the response to be tested. Three ages would have been desirable, but the experiment was already rather large for the facilities available, so only two ages were used. Thus the experiment required $4 \times 4 \times 3 \times 2 = 96$ observations. If three ages had been used, the number required would have been $4 \times 4 \times 3 \times 3 = 144$, which was considered too much.

This business of the choice of numbers of levels is almost always troublesome, because our facilities are rarely equal to our ambitions. We have to make compromises; sometimes they will turn out satisfactorily, other times we will regret our choices. But designing an experiment is always partly guesswork; if we knew enough to be *sure* we have a satisfactory design, then we usually know so much we hardly need to perform the experiment.

At first sight the experiment in Table XIV appears to be a simple four factor factorial; of course it could be if we so desired. An appreciable economy in facilities is possible, however, if we adopt a split-plot design.

The Split-Plot Situation

The split-plot situation was first recognized in agricultural experimentation. Here it is sometimes difficult to sow very small plots with different varieties of, e.g., corn. On the other hand, different fertilizer treatments can be comparatively easily applied to small plots. Thus, a field experiment might have the layout in Table XV. This represents the basic unit and needs randomization and replication.

The single lines denote the margins of the whole plots, containing four subplots all sown with the same variety v but receiving different fertilizers f_1, f_2, f_3, f_4. This design is structurally different from a randomized block design, where we would number the twelve subplots 1 to 12 and allocate the twelve treatment combinations v_1f_1, etc., to them at random.

It will usually be the case, if all treatments were identical, so that all differences in yield were due to error, that the differences between adjacent plots will be smaller, in general, than differences between remote plots. Thus in the split-plot design the error appropriate to the comparison of the variety means will

TABLE XV

v_1f_1	v_1f_2	v_2f_1	v_2f_2	v_3f_1	v_3f_2
v_1f_3	v_1f_4	v_2f_3	v_2f_4	v_3f_3	v_3f_4

TABLE XVI

Randomized Block		Split-plot	
Source of Variance	Degrees of Freedom	Source of Variance	Degrees of Freedom
Replicates	(r−1)	Replicates	(r−1)
Varieties	(v−1)	Varieties	(v−1)
Fertilizers	(f−1)	Varieties × Replicates (Error I)	(v−1) (r−1)
Varieties × Fertilizers	(v−1) (f−1)		
		Fertilizers	(v−1)
Error: made up of		Fertilizers × Varieties	(f−1) (v−1)
		Error II: made up of	
R × V	(r−1) (v−1)	R × F	(r−1) (f−1)
R × F	(r−1) (f−1)		
R × V × F	(r−1) (v−1) (f−1)	R × F × V	(r−1) (f−1) (v−1)
summing to	(fv−1) (r−1)	summing to	v(f−1) (r−1)
Total	vfr−1	Total	vfr−1

be larger, since this involves subplots relatively far apart, than the error appropriate to the comparison of the fertilizer means, since this involves plots immediately adjacent. The analysis of variance (Table XVI) must take account of these differences.

Table XVI shows that for the split-plot design we have two error terms, each appropriate to a particular part of the analysis. Usually Error I will be greater than Error II; it can only be really less in very peculiar circumstances.

The split-plot situation occurs very often in industrial work, but the necessity for this type of analysis is often overlooked.

The Analysis of Our Example

In our fermentation experiment, we choose to obtain the observations at different ages *not* by running another fermentation to a different age but on the contrary by taking samples at two different ages out of the *same* fermentation. Age now becomes a subplot factor, and the analysis of variance is as in Table XVII.

Table XVII illustrates another aspect of the design of factorial experiments; if we have a multi-factor experiment, we often can rely upon using a relatively high order interaction (here Batch × Strain × Concentration) as our estimate of error, and we can dispense with explicit replication to provide us with an estimate of error. Explicit replication sometimes, of course, may be necessary to give sufficient accuracy.

We can rapidly draw a number of useful conclusions from Table XVII:

(a) The Batch × Strain interaction is not significant. Therefore we may accept the batch differences as being independent of strain.

(b) The Batch × Concentration interaction is not significant. Therefore we may accept the batch differences as being independent of concentration.

(c) In the subplot part of the analysis there is some slight evidence that the factor batch might be involved in a high order interaction, Age × Batch × Concentration. The evidence is not strong, however, and in general high order interactions are troublesome to interpret, so we will not pursue this. Similarly the evidence for the Age × Batch interaction is weak and we will take it to be nonsignificant.

(d) We therefore conclude that within our experimental error the factor batch is independent of the other three factors, concentration, strain, and age, and so we may look at the main effect, which is significant at the 0.005 level. We would therefore look at the batch means and compare them using the appropriate standard error.

(e) The above completes our main interests, namely a comparison between batches. We have gotten at the same time some other results which are of interest.

(f) There is strong Age × Strain interaction, shown in Table XVIII.

The 95 percent confidence limits for the difference between the means for the two ages for a given strain are ±5.9. It is clear that the strains are different in their time-yield curves. We regret the decision to use only two ages.

TABLE XVII

Source of Variance	Degrees of Freedom	Sums of Squares	Mean Squares
Batches	3	2498.083	832.694
Concentration			
L	1	24492.250	24492.250
Q	1	196.021	196.021
Strain	3	11601.250	3867.083
Batch × Concentration			
L	3	55.875	18.625
Q	3	185.354	61.785
Batch × Strain	9	797.000	88.556
Strain × Concentration			
L	3	307.250	102.417
Q	3	449.062	149.687
Batch × Strain × Concentration (= Error I)	18	1976.187	109.788
Age	1	2166.000	2166.000
Age × Batch	3	329.750	109.917
Age × Concentration			
L	1	289.000	289.000
Q	1	0.188	0.188
Age × Strain	3	3905.583	1301.861
Age × Batch × Strain	9	823.000	91.444
Age × Batch × Concentration			
L	3	322.125	107.375
Q	3	360.687	120.229
Age × Strain × Concentration			
L	3	561.000	187.000
Q	3	278.730	92.900
Remainder (Error II)	18	860.938	47.830
Total	95	52455.333	

A split-plot experiment has the peculiar feature that a two-way table, such as Table XVIII, has different confidence limits for comparing the means in different ways. If we wanted to compare two strains, at the same or different ages, the 95 percent confidence limits are ±9.0. The two errors, of course, arise because the comparison of the previous paragraph, between ages for the same strain, are on a *within* whole-plot basis, whereas the comparisons mentioned here are on a *between* whole-plot basis.

(g) There is probably an Age × Concentration interaction (Table XIX). It is unimportant compared with the effect discussed earlier. It suggests that the richer medium is relatively quick so approaches its maximum more closely at the earlier age than the weak medium.

(h) As anticipated, Error II, within fermentations, is smaller than Error I, between fermentations.

TABLE XVIII

	Strain A	B	C	D
Age A_0	58.7	68.6	76.5	86.9
A_1	69.6	78.9	102.7	91.5
A_1—A_0	10.9	—9.7	26.2	4.6

Summary

The split-plot situation may arise from choice or by the nature of the process. In the present example, if our main purpose had been to investigate the effects of age we would have chosen this design, because we measure the effects of age on a within whole-plot basis which is more accurate. On the other hand, if the main cost, or bottleneck, of the experiment had lain in the microbiological analysis of the individual samples, so that the cost was the same whether we had two ages on each of 48 fermentations or one age on 48 fermentations and the other age on another 48 fermentations, and if our main objective was the comparison of batches, then it would have been better to run the experiment the second way (i.e., with 96 fermentations), because we would then have had more replicate determinations on each batch.

This example has been intended to illustrate the following points:

(a) the need to investigate factors factorially so as to detect interactions. As it turned out, for the factor batch the interactions are not significant, so this was an unnecessary precaution. On the other hand, for the factor strain interactions are very important and overlooking them could have given us a completely false picture;

(b) the difficulties in compromising between our desire for a moderately large number of levels and our facilities. As it turned out, we regret using only two levels for the factor age;

(c) the difference a split-plot structure makes to the analysis of variance;

(d) the fact that a split-plot situation introduces different standard errors in the two-way tables;

(e) the usage of high order interactions as estimates of error instead of using explicit replication.

Conclusions

The over-all conclusions are contradictory. On the one hand one is tempted to conclude that all we need for good experimental design is a clear head and that anyone should be able to design a good experiment. On the other hand, we repeatedly encounter situations which even with years of specialized experience and the whole vast bag of tricks we find very difficult to handle. In addition, there are a number of troublesome problems in the analysis of variance for which there are no easy answers known to us.

I can only recommend that you should read Fisher's *Design of Experiments*[11] over and over again. Even on the 21st reading you will still be getting fresh understanding out of it. Kempthorne's *Design and Analysis of Experiments*[12] is equally worthy of study. The subject is so rich that one can easily spend a week absorbing the implications of a single section in either of these texts. One overriding feeling I have is that often we try to do too much with too little. In our research work I often yearn for a little more care and craftsmanship, a little less haste, a little more thoroughness, a little more thought and a little less impetuous leaping around. The shoddy and inadequate experiment may often get us the right answer, as often by good luck as by good judgment, but I think that we would be better off in the long run if we did a more thorough job. Granted that this

TABLE XIX

	Concentration C_0	C_1	C_2
Age: A_0	59.2	79.6	94.1
A_1	79.6	89.2	107.8

may take a little more time, it is only because the work should have been started six months ago that we are in such a hurry today. So let us plan the strategy of our research more carefully; then the tactical problems will be much easier.

References

(1) Anscombe, F. J., "The validity of comparative experiments," *Journal of the Royal Statistical Society, A, 61,* 181-211 (1948)

(2) Bartlett, M. S., "The use of transformations," *Biometrics, 3,* 39-52 (1947)

(3) Bose, R. C., "On the application of Galois fields to the problem of the construction of Hyper-Graeco-Latin Squares," *Sankhya, 3,* 323-338 (1938)

(4) Stevens, W. L., "The completely orthogonalized square," *Annals of Eugenics, 9,* 82-93 (1938)

(5) Davies, H. M., "The application of variance analysis to some problems of petroleum technology," *Journal of the Institute of Petroleum, 32,* 465-491 (1946)

(6) Yates, F., "Incomplete randomized blocks," *Annals of Eugenics, 7,* 121-140 (1936)

(7) Youden, W. J., "Experimental designs to increase accuracy of greenhouse studies," *Contributions of the Boyce Thompson Institute, 11,* 219-228, (1940)

(8) Frazier, D., Klingel, A. R., and Tupa, R. C., "Friction and consumption characteristics of motor oils," *Industrial and Engineering Chemistry,* 45, 2336-2342 (1953)

(9) Eisenhart, C., "The assumptions underlying the analysis of variance," *Biometrics, 3,* 1-21 (1947)

(10) Johnson, N. L., "Alternative systems in the analysis of variance," *Biometrika, XXXV,* 80-87 (1948)

(11) Fisher, R. A., *Design of Experiments,* Oliver and Boyd, Edinburgh, 4th Edition, 1947

(12) Kempthorne, O., *The Design and Analysis of Experiments,* John Wiley, New York, 1952

SUPERVISOR'S CIRCLE

PAUL A. ROBERT, *Editor*

Gruen Watch Company, Time Hill, Cincinnati, 6, Ohio

Calibration Control of Inspection Gages

MARTIN W. JOSEPH AND FRED W. VOGEL, *Hughes Aircraft Co., Tucson, Ariz.*

Comments of the Editor

The Air Force Specification MIL-Q-5923C states "the contractor shall provide and maintain suitable gages and other measuring and testing devices used for inspection necessary to check supplies for conformance to contract requirements. Such devices shall be checked with suitable measuring equipment at established periods to assure continued accuracy. The contractor shall establish a schedule of such checks as a portion of his inspection procedure based on type purpose and degree of usage and shall maintain records or other conclusive evidence that proper control is being provided."

This is the system currently in use at Hughes Aircraft Company.

The editor solicits the comments of our readers and descriptions of other gage control systems currently being used by Air Force contractors.

How to control calibration of measuring equipment in a manufacturing operation which requires more than 25,000 production and inspection gages was a major problem until someone found a simple solution. It was not done with mirrors, but with something much less destructible and more attractive, that is, if you like colors.

Two colors were added to the handle of a gage, or some other convenient spot where the application of the color would not interfere with the gaging operation. In some cases where the steel was sufficiently soft, a niche was made with the corner of a file; in other cases small holes were drilled or where the steel was extremely hard, the two painted dots were placed on the surface, preferably where a serration existed.

The person who recommended color codes for calibration frequency identification had heard something about combinations and in checking

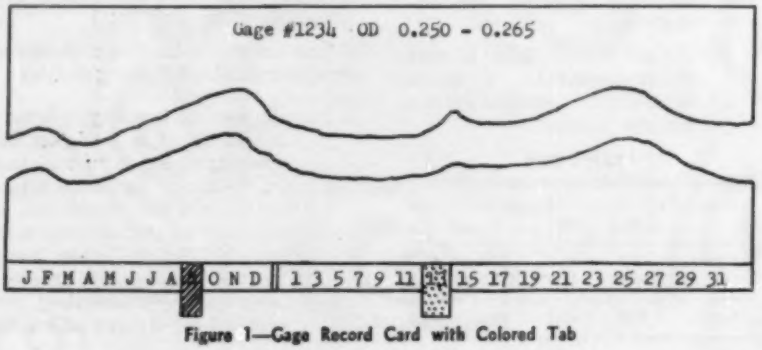

Figure 1—Gage Record Card with Colored Tab

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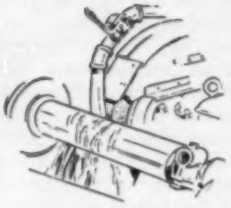
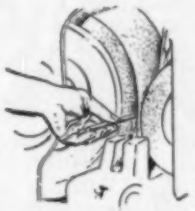


CYLINDRICAL GRINDING

A Wisconsin plant, grinding 2" to 4" diameter steel shafts, 12" length, reports 44 ALUNDUM wheels (44A54-KVBE) cut faster and cooler with no sacrifice of wheel life, compared with best previously used wheels.

A Michigan plant, grinding miscellaneous high speed steel bolt heading dies, finds 44 ALUNDUM wheels best. Reasons are more pieces per dressing, cooler cutting action and 30% longer wheel life than formerly used wheels.

A New Jersey plant found a 44 ALUNDUM wheel (44A36-MVBE), plunge grinding different steel parts, gave many more pieces per dressing and averaged 15% longer life than wheels they used before.



CENTERLESS GRINDING

A Michigan plant, of the smaller jobbing types, grinding all kinds of materials. They prefer 44 ALUNDUM wheels (44A801-NVBE) for versatility, less dressing and faster cutting — including 10-hour completion of a job which used to take 14 hours.

An Ohio plant removes .008" on the rough grind and .003" on the semi-finish grinding of No. 1010 steel crankshaft pins. While standard wheels gave an average of 600 pieces per dressing, they report a 44 ALUNDUM wheel produced 1,000 pieces. All stock was removed in one pass, compared with two passes formerly required.

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TABLE I—Gage Calibration Color Code

Frequency of Calibration	1st Color	2nd Color Denotes Period Through Which Gage May Be Used																					
		Jan.		Feb.		March		April		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.	
Semi-Monthly	RED	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
		R	Bl	O	G	Br	P	R	Bl	O	G	Br	P	R	Bl	O	G	Br	P	R	Bl	O	G	Br	P
Monthly	BLUE	R		B		O		G		Br		P		R		Bl		O		G		Br		P	
Quarterly	ORANGE	R						B						O						G					
Twice Yearly	GREEN	Odd Years					R							Bl											
		Even Years					O							G											
		Code: R-Red Bl-Blue O-Orange G-Green Br-Brown P-Purple																							

found that with six (6) colors, using two (2) colors at a time, fifteen (15) different arrangements can be made. Thus the following periods were identified; 24 semi-monthly periods per year; 12 monthly, 4 quarterly and 2 semi-yearly. The division of gages into these calibration frequencies was based on the functional requirements of the gage, the required gage accuracy, and the past history with respect to gage wear. All Production and Inspection gages were given a 2-color code designation according to the procedure shown in Table I. To use the table, the inspector notes the first color mark. This indicates the period of check. For example, if the first color mark is Red, the gage is on a semi-monthly schedule. If the second color mark is Blue, and today's date is February 16, the gage is to be returned for recalibration by February 31.

The format in Table I was printed on a card and issued to every inspector who is now required to check whether or not gages are due for calibration before the gages are used for acceptance inspection. It should not be inferred that we rely completely on the inspector in calling out when calibrations are to be performed. The Kardex System, briefly described in the next paragraph, takes care of that. Please note that the use of 8 instead of 6 colors on the above table would have assured that no combinations need be repeated. However, 6 colors are easier to handle than 8 and repeat combinations were spaced sufficiently far apart so as not to be confusing to the inspector.

The Kardex system described below makes it easy to identify what particular gage is to be calibrated. Again, a color code has been devised to denote the calibration cycle, that is, semi-monthly, monthly, quarterly, and twice-yearly. Within these groupings a tab, as shown in Fig. 1 attached to the bottom of the gage record card, identifies the specific month and day range for calibration of the gage.

This is what happens when a gage is received for the first time at the Calibrations Laboratory:

1. It is checked and the dimension found is recorded on the stage history card.

NOMINATIONS FOR THE 1955-1956 BRUMBAUGH AWARD

In accordance with the provisions governing the Brumbaugh Award, nominations by individual members of ASQC are requested in order to assist the Brumbaugh Award Committee in making its selection. Below is a ballot for your use. Please fill in the ballot and *mail it now.*

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Send your ballot to Cuyler J. Hawkes, Eastman Kodak Company, Camera Works, Rochester 4, New York. All Ballots received up to March 15, 1957 will be accepted.

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The articles which are eligible for this year's award are those that appeared in Volume XII of IQC—July 1955-June 1956.

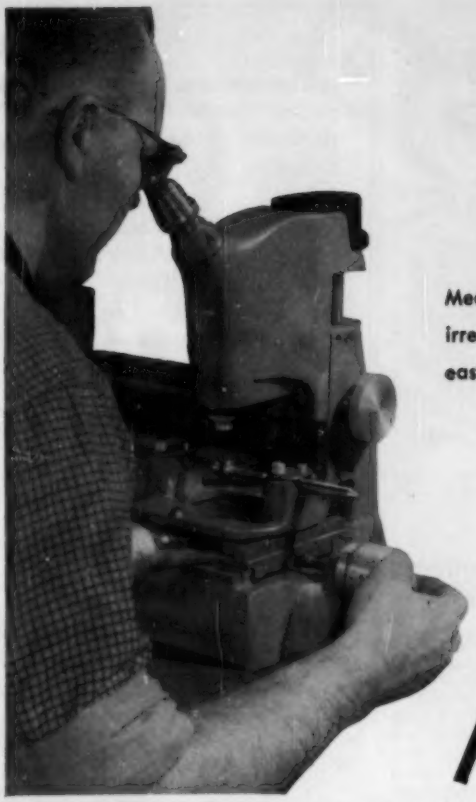
2. Gage wear tolerances are entered on this card.
3. Calibration frequency designation is made.
4. Next calibration is shown on the gage record card according to the procedure shown on the tab in Fig. 1.

The appropriate changes with respect to Steps 3 and 4 above are made after each calibration. If out-of-calibration conditions are found too frequently, a change in calibration designation is made and the gage placed into a shorter calibration period. This system has proved to be quite successful because of its ease of operation.

What's New?

ROY A. WYLIE, *Editor*

The **P. A. Sturtevant Co.**, Addison, Ill., manufacturers of torque wrenches, announces a full range industrial production torque wrench tester. The new precision torque tester features a torsional shaft measuring element that is submerged in pre-heated oil to enable constant operating temperatures for the measuring device. There is no mechanical connection between the measuring element and the indicating pointer, as the pointer is projected onto the scale through an optical system. The indicating pointer travels across a ribbon scale for a distance in excess of ten feet in total length. It is a shadow image and therefore there is no parallax error when an operator views the screen. The Optical Torque Tester is equipped with a power loading device, thus the operator will no longer be confronted with the fatigue of handling torque wrenches or the delay encountered in loading with hand-operated mechanical devices. The movement of the loading arm can be controlled with a metering valve so that it will move as slowly as the hands on a clock or as rapidly as need be to load any type of torque wrench to full capacity almost instantly. Provision is made on the cabinet of the Optical Torque Tester to permit mounting special external fixtures for miscellaneous production



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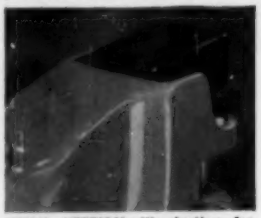

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EDITORIAL NOTE

The authors, Mr. F. Akutowicz and Mr. H. M. Truax, of the paper, "Establishing Control of Tire Cord Testing Laboratories," published in the August 1956 issue of IQC, have requested that the following note be brought to the attention of readers of this journal:

"I wish to point out to readers of IQC that the method of obtaining Expected Values of Mean Squares which we used in our paper is not a satisfactory alternative to the correct method, which has been outlined recently by Professor Hicks. The fact that the numerical results of the experiment are the same either way is only a coincidence, and does not justify continued use of the older method."

The reference to Professor Hicks concerns his three papers on "Fundamentals of Analysis of Variance" which were published in the August, September, and October 1956 issues of IQC, and are available in complete reprint form from the Milwaukee office at 45 cents per copy.

M.E.W

and laboratory tests, such as the test of a sensitive clutch mechanism, etc. With a simple fixture this equipment also lends itself to the recalibration of strain-gage devices that are frequently used for special field tests. Another adapter is available as a stock item for recalibrating cable tensiometers. The Optical Torque Testers are available in three capacity ranges: 0 to 250 inch-pounds; 0 to 165 foot-pounds; and

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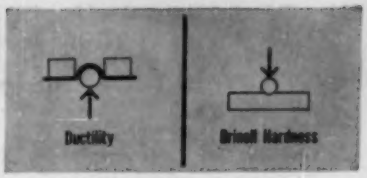
0 to 500 foot-pounds. Accessories for the most used types of torque wrenches are contained in the tester cabinet. This equipment has previously been made on special customer order, but with increased demand they are now expected to be available as standard production models and stock deliveries may be made soon.

★ ★ ★

A new electronic inspection unit to check the contents of packages has been introduced by the **Photobell Sales Corp.**, 43 Vesey Street, N. Y. 7, N. Y. The new package inspector is based on the use of high-frequency current passing through the entire package. All materials respond to this current in proportion to their own electrical properties, as well as to the volume of the materials present. High-frequency current will usually permeate right through the package itself, measuring the quantity of most materials by allowing current to flow in proportion to the amount of material present. Since some materials conduct electrical current more efficiently than others, it is also possible to single out certain materials inside a complicated package, and determine if they are present in the wanted quantity, ignoring other materials in the package. While this cannot be done in all cases, it is frequently possible and greatly simplifies many control operations. The package inspector can be used to sort and remove from the conveyor all packages which have less than the proper number of items or to remove packages which are slightly underweight. It may also be used to remove packages which do not fill up to the wanted appearance level regardless of weight or to remove those cartons which do not have the correct number of bottles or in which the bottle is not quite full. It may also be used to detect whether the package contains a premium item. Polethylene squeeze bottles can be checked to insure the proper level of liquid in the bottle. Many other uses in package inspection are possible. Principal categories of products which cannot be checked in this manner are metal containers which are closed on all sides; non-uniform packages for such items as bananas and fruit; and randomly distributed packages such as bags of hardware.

★ ★ ★



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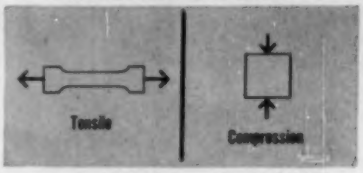

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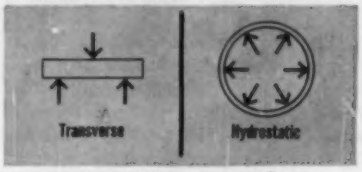

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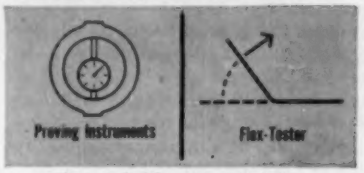

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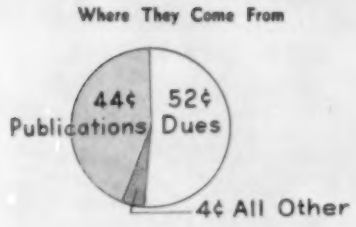
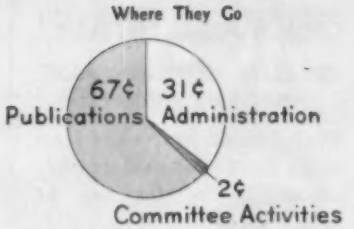


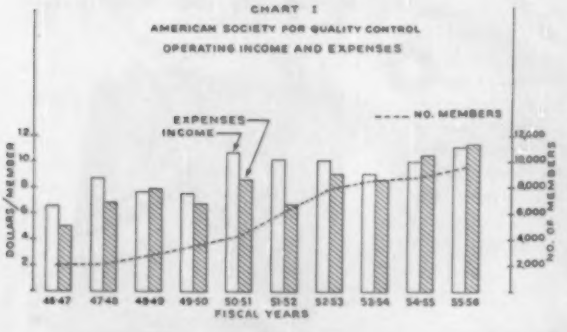


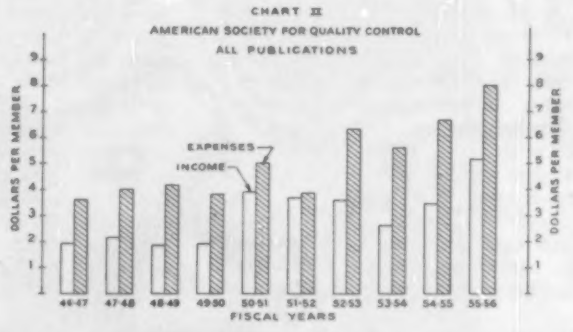
Financial statements and details of receipts and disbursements of operating funds of the Society are published regularly in the September issues of *Industrial Quality Control*. No doubt many members of the Society have reviewed these figures from time to time as a matter of interest and are acquainted with the Society's financial operations. In general, however, pages of figures available once each year fail to reflect over-all trends in matters bearing directly on the financial stability of the organization.

At the risk of over-simplification, the divisions of the Society's income and expenses for 1955-56 are illustrated above by the familiar "pies", showing relative amounts in each of the major categories. The pattern for expenses has remained essentially constant in recent years, whereas the dues portion of the income dollar was substantially lower last year than in the past five years. Dues income in total dollars has, of course, increased with our growing membership; at the same time publications income (advertising, sale of technical literature, and institutional subscriptions to IQC) has increased at a somewhat greater rate.

A quick glance at Chart I shows more clearly the changing financial position of the Society with respect to operating income and expenses. Whereas income per member has not changed appreciably over the past six years (since the last dues increase), expenses have increased until in the past two years they have exceeded income.

Our income from publications on a per member basis, increasing as it has in the very recent years, represents an increasing percentage of the total income. Nevertheless, as a result of increasing our services in the publications area—such as putting out IQC on a monthly schedule since January 1955—publications expenses have increased steadily over the years, as shown in Chart II.





BOOK REVIEWS

W. D. Baten, Editor

26 University Drive, East Lansing, Mich.

CURSO DE ESTADISTICA, Vol 1 by Enrique Chacón, published by Editorial el Mensajero del Corazón de Jesús, Apartado 73, Bilbao. 1955. XVI plus 494 pages. Reviewed by W. D. Baten.

This contribution to statistics, in Spanish, was written for students who are familiar with advanced mathematics, for a great deal of the material is based upon the theory of real variables, the theory of equations, matrix theory, n-dimensional geometry, theory of probability and other subjects developed in higher mathematics.

Generating functions are used throughout the text for finding the moments of various frequency distributions in one and more than one chance variable. After developing the binomial, the Poisson and the hypergeometric distributions, the author shows that they approach the normal curve when the number of trials becomes large. Moments for various statistics are obtained.

The normal distribution is adequately explained together with probability paper, tables for this curve and derivations of its parameters. Distributions in two and more variables are very interestingly developed together with their moments.

Chapters on correlation coefficients are well presented, especially multiple and partial correlation; here is shown by an example how the complicated correlation coefficients obtained by building on to the simpler ones.

Among other topics, the reader will find ideas concerning: characteristic functions, mean of Gini, percentiles, functional paper, incomplete beta functions, standard error of estimate, discriminant functions, Tchebychef's inequality, intraclass correlation, tetrachoric correlation, biserial correlation, ellipse of concentration, circular probability paper, net regression, logistic curves and correlograms.

Sampling theory is introduced in which is described in many details the expectations of the moments of various statistics. Statistical theory concerning time series is illustrated in three chapters which indicate the purposes of secular trends, seasonals and cycles with several diagrams.

The author does a good job in the development of the distributions of the following: averages, sums of squares, Chi-square, differences of averages, Student's-t, regression coefficients, the correlation coefficient, Snedecor's F, Hotell's T^2, Chi, difference between 2 regression coefficients, median, etc.

At the end of each chapter are well chosen exercises for the student to solve that should help in his development and enable him to understand the material presented.

At the end of this book are the following tables: normal curve, Chi-square, t-distribution, F-values, Poisson distributions, range distribution, w_1/w_2, values of 2 arc sin $\sqrt{X}$, binomial paper, tables of the distribution of L_0 & L_1, the first four powers of integers, factorials, log of factorials, and square roots.

I am glad to have this well written book in my library; it is one of our better second courses in statistics because it contains so many of the fundamentals of statistics and also because it is so easy to read.

A HUMAN ENGINEERING BIBLIOGRAPHY, by Ivan N. MeCollom and Alphonse Chapanis, published by San Diego State College Foundation, San Diego 15, Calif., Nov. 1956. 128 pages. paperbound. 8½ × 11. Reviewed by N. L. Enrick.

Close to six thousand articles on human engineering design are listed, classified by sixteen major categories and close to 100 sub-headings. Design of equipment for human use as well as setting up of man-machine systems were considered in selecting articles for inclusion. Categories which may be of special interest in quality control applications are (1) man-machine systems, (2) visual problems and (3) control systems. Subject and author indexes were not included.

Local Section News

"... BUT NOT FORGOTTEN"

ALBUQUERQUE "The use of attributes for measurements of continuous variables in the rocket industry" was discussed by O. R. Weaver, quality control manager of Phillips Petroleum Co., Mac Gregor, Texas, at the Dec. 10 meeting. The educational session which preceded Mr. Weaver's speech was led by J. C. Connell, Sandia Corp., and consisted of a review of a paper by H. F. Dodge and M. N. Torrey entitled "A check inspection and demerit rating plan" which appeared in the July 1956 issue of IQC.

ALLENTOWN -BETHLEHEM The December meeting consisted of a conducted tour through the Parish Pressed Steel Co., Reading, Pa. Prior to the tour, many of the participants met to dine and chat in the International Room of Leo's Restaurant, in the environs of Reading.

BALTIMORE The December meeting was held at the Johns Hopkins University. George Georgis, who was guest speaker, spoke on the subject "Process capability studies" to the group of approximately 60 members and guests.

BOSTON A good attendance—60 members and guests—was present at the Dec. 5 meeting. Following the practice of the last several years to have one meeting on an operations research topic, John Burlingame of the General Electric Co., discussed OR and its applications to business.

The standard orientation course on quality control, presented annually by the section, was completed on Dec. 13. Although the course was briefer this year than previously, the 25 registrants maintained an unusually high degree of interest throughout.

BUZZARDS BAY Charles A. Reed, quality control manager of the clock and timer department, General Electric Co. in Ashland, was the main speaker at the Dec. 11 meeting held in New Bedford. His subject was "Visual inspection procedures." A dinner for Mr. Reed at the New Bedford Hotel preceded the meeting. Illustrating his lecture with charts, Mr. Reed outlined methods for eliminating the guesswork and opinion in visual inspection procedures. He demonstrated the use of control charts and sampling plans, citing cases from the manufacture of timers used with radios, bank vaults, and traffic lights.

CHICAGO At a recent board of directors meeting held at the Wahl Henius Institute, discussions were held regarding consideration of the importance of welcoming new members, as well as maintaining interest of older members. Plans for a social evening were presented by program chairman E. G. Freund. Board members were requested to make inquiries before their next meeting, to determine the extent of anticipated attendance for a dinner-dance

type of event. Chicago section's policy of supporting sub-groups was also explained at this occasion.

"Attribute charts" was the topic of the section's December meeting. A group of 46 attended this interesting session, led by Fred Chiarlotte and Dale Gierke. The committee in charge of these programs wants to remind residents of the Chicago area who may have missed this meeting or other earlier portions of the program that it is still possible to participate in the remaining sessions and to benefit from them.

CINCINNATI The Dec. 12 meeting was held in collaboration with the Cincinnati section of the American Chemical Society. David R. Peryam of the Quartermaster Food and Container Institute in Chicago was the principal speaker. The talk on the "Human factors in the sensory evaluation of products" was very interesting and was well received by the group. Chairman Birch considered the joint session very successful and cited it as an example of inter-society relationship.

National vice president Leon Bass spoke before the Cincinnati section of the Institute of Radio engineers on Dec. 18. Mr. Bass' subject "Design for quality and profits" discussed the elements of a design engineers' job and how common sense, quality control techniques, and statistical method can supplement the engineers' technical skill and judgment and enable him to produce better designs in shorter time at less cost.

The December meeting of the Hamilton-Middletown subsection was held at Pillsbury Mills with H. J. Jacobson, quality control manager of P. R. Mallory Co. of Indianapolis, as the speaker. An informative and entertaining talk was given by Mr. Jacobson on how quality control may help on purchasing.

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The Glenn L. Martin Co., builders of such interesting projects as the EARTH SATELLITE and the INTERCONTINENTAL BALLISTIC MISSILE, has excellent openings for Statisticians.

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CLEVELAND Society vice president Leon Bass, manager of quality control for the Jet Engine Department of the General Electric Co., was speaker at the Dec. 7 meeting. Mr. Bass delivered a talk entitled "Make it right the first time" which showed that improved quality and cost reduction can result from the simplest of techniques. Defect prevention is stressed as an objective rather than defect rejection.

CORNING -ELMIRA Fifty members and guests of the section met Dec. 11 at the Hotel Langwell, Elmira, and were honored to hear Blair E. Olmstead, research analyst for the Prudential Insurance Co., Newark, N. J. speak on "Clerical quality control applications."

The first part of the section's educational program concluded on a very successful note. Sixty-four people, including 41 non-members, registered for the course, entitled "Elementary statistical analysis." Instructors for the course were Professors R. Bechhofer and J. Kao.

DALLAS -FORT WORTH O. R. Weaver presented "Using attributes for measurement of continuous variables in the rocket industry" at the Dec. 13 meeting held at Arlington State College. Mr. Weaver stated that by the use of the Bruceton Staircase method data, such as impact, sensitivity, exposure to detrimental elements, or similar inspection tests of a destrucitve nature, can be arranged in a manner which provides a mean and standard deviation and thus can be converted to variable inspection. Reliability much beyond the corresponding figures when attributes alone are used are obtainable in this manner at much reduced cost. Mr. Weaver is currently the

chairman of the Waco section and is manager, quality control branch, Rocket Fuel Div., Phillips Petroleum Co.

DANVILLE -SUNBURY The December meeting was held in the form of a plant tour of the American Home Foods Co., Milton, Pa. The tour was conducted by the quality personnel of the company to acquaint the ASQC members, wives, and guests with the "pizza dinner" and "spaghetti" lines. A buffet luncheon was served after the tour.

DAYTON A social meeting was held on Dec. 13. This was the first time in several seasons that this type meeting was held. Due to the holiday season it was felt that a social meeting would receive better attendance than a technical one. Those in attendance had the pleasure of seeing two short movies; one on the fabrication of stainless steel titled "An orchid for Mr. Jordan" and a movie on wildlife preservation. This meeting was held at the Delco Lodge with refreshments being served by the section.

Twenty-six people successfully completed the basic training course offered by the section. This course was completed in December. The instructors for the course were all members of the section.

DELAWARE About 35 members and guests attended the dinner meeting on Dec. 6 at the Lord De La Warr Hotel in Wilmington. Meredith E. Smith of Du Pont's engineering department gave a most interesting and instructive talk on an operations research subject, "Some practical applications of Queuing models." "Smitty," a member of the section, gave several examples of industrial waiting lines which can include personnel or material or a combination of both at various stages of an industrial process.

ERIE Our speaker at the Dec. 4 meeting was Charles F. Bicking of the Carborundum Co. of Niagara Falls. Mr. Bicking's topic was "Design of experiments." The dinner-meeting was held at the private dining room of the Hotel Kenyon. Attendance was down compared to October and November meetings, but this seems to be a chronic condition every December.

On Dec. 11 the officers held the monthly luncheon at Shugarts. Cancellations by two of the speakers forced a revision of our announced spring program. Substitutes were to be contacted.

GEORGIA At the Dec. 18 meeting of the Atlanta group, Dr. George M. Brown of the department of mathematics at the Georgia State College of Business Administration, presented an excellent talk on "Some theory of runs." He explained the conventional symbolization and then went into the mathematical equations. We hope to hear him develop the permutations later.

GREATER MUSKEGON A record attendance of 40 members and guests was set at our Dec. 13 meeting. J. E. Rutledge, chief inspector of the McConnel Aircraft Corp. of St. Louis, was guest speaker. Mr. Rutledge's topic was "Quality control without statistics." Slides of McDonnel's inspection system added to the presentation.

Mike Pascavis, program chairman, announced that A. Lindberg, E. Hansen, J. Yates, and Bert Taulbee would serve as panel members at the next meeting which would be a fish fry at the Mona Lake Boating Club. The panel topic will be on "Salvage operations."

HAMILTON T. P. Monaghan of the Ford Motor Co. of Canada spoke on the subject of "Sampling Plans" at the Dec. 13 meeting held in the Canadian Westinghouse auditorium. He discussed the statistics underlying a good sampling plan and the advantages of such a plan over the old "spot check." A comparison was made of "single lot" sampling, "double lot" sampling, and "sequential" sampling.

HARRISBURG The first annual ladies night was held at the American Legion in Elizabethtown on Dec. 5. A new building, pretty Christmas decorations, the usual excellent talk by Dr. Ellis Ott, and last but most important of all, the presence of the ladies contributed to make this one of the most successful meetings of the section. Many of the industries represented in the section contributed small gifts for door prizes to add to the occasion.

Dr. Ott provided the men with much food for thought with his talk on "Concept of a specification" while at the same time Mrs. Ott showed the ladies a number of their slides taken in India during their United Nations quality control mission. (Dr. Ott is pictured above with chairman Miles Nolte on his left, and program chairman Willard Smith on his right.

KANSAS CITY Approximately 100 members, guests, and their women frolicked at the December social event. A buffet lunch was served at the Homestead Country Club with dancing to the tune of canned music. Mixer games enlivened the activity.

KITCHENER A group of forty odd members and guests toured the General Motors Diesel plant in London, Ont., on Dec. 12. Groups of five or six with a mechanical or electrical background were formed and given qualified guides. The group was then given four problems to think about and observe during the tour that would be discussed later. The group was greatly impressed by the immensity of the operation and ingenious methods used in assembly.

LANSDALE The December meeting was held at the Eagles Hall. Our speaker at this meeting was Dr. Howard Steir of the National Canners Assoc., Washington, D. C. After being introduced to the group by section president Harry Bear of Hunter Spring Co., Dr. Steir spoke of the growing need for statistical understanding and knowledge by the average citizen, the use of statistics by management as a basis for decisions instead of using hunches, and the need for effective recording and presentation of data. Visual aids were used throughout the talk. After the meeting was

adjourned an informal discussion was held during which refreshments were served.

LINCOLN The Dec. 14 meeting was held at "Marchio's" in Omaha. This meeting was designated as "Ladies' Night" in honor of the wives of the members. In spite of threatening weather, 42 members and guests were present to hear Kieth Schroeder of Western Electric Co., Lincoln, give an interesting talk entitled "Statistical quality control for the wives." A film presentation of the J. C. Penny Co. of Omaha, "We use quality control this way," was enjoyed by all.

LOS ANGELES The December meeting discussed acceptance procedures and vendor-vendee quality control relationships. Gordon Bowler, QC manager of Hughes Aircraft Co., El Segundo, described the work done by RETMA, a trade association in the field of electronics, to clarify and standardize such practices. A consistent, objective plan, based on both engineering and quality control judgment, recently instituted in Northrop Aircraft, was described by Jack Mannion, staff assistant to vice president and general manager of that company. It was stressed that while QC as a whole is coming of age, vendor-vendee relations have not been as enlightened and consistent as they should be. Work done in this area will force vendor and vendee to coordinate and standardize their inspection characteristics and test procedures and will contribute to reduced inspection.

LOUISVILLE The Louisville chapter of the American Institute of Industrial Engineers and the section enjoyed a joint dinner meeting at the American Radiator and Standard Sanitary plant on Dec. 10. E. M. Schrock, a "missionary" for quality control in its most practical applications, spoke on the "Organization of an efficient quality control program." We were proud to present a known speaker of Mr. Schrock's experience and enthusiasm at this joint meeting.

Twenty-five persons now hold certificates for the satisfactory completion of the advanced quality control and data course presented by the Louisville education committee and instructed by Burr Stanton and A. Greene, Jr.

MICHIGAN Poor weather conditions prevented Blair E. Olmstead of Prudential Insurance Co. of America from appearing at Wayne University in Detroit last Dec. 14. Mr. Olmstead, a research analyst, was to have discussed "Clerical quality control applications." The Detroit chapter of the National Office Management Association and the Michigan section were deprived of what promised to be an enlightening presentation.

Dr. Eugene C. Yehle, School of Business Administration of the University of Michigan, saved the evening with an impromptu discussion of the same subject.

Raymond L. Clark, Argus Cameras, Inc., Ann Arbor, delivered a truly clear picture of "multiple correlation" at the advanced educational program which preceded the regular meeting.

Harry L. Wehrly, quality control manager of the Dow Chemical Co., was guest speaker of the Wolverine subsection at the Dec. 7 meeting. Mr. Wehrly's presentation on "Planning and organizing for quality" was very interesting. The session was held at the UpJohn Co., Kalamazoo.

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MID-HUDSON C. K. Hamlin, quality manager of the Rome Cable Corp., spoke on "Quality control in the wire manufacturing industry" at the December meeting. The use of statistical quality control techniques during processing has reduced final inspection at Rome Cable to the point that government inspection is administered through RIQAP—reduced inspection quality assurance program.

MILWAUKEE The ladies always enjoy the special night set aside for them. This year, the Dec. 10 cocktail party, dinner, and the entertainment after dinner was one of the most successful that we've had.

MINNESOTA Dr. Charles R. Hicks of Purdue University addressed the section Dec. 11 on the subject of "Statistics applied to industrial engineering."

MONTREAL The monthly meeting of the section was held at the Mechanics Institute. James Eastcott of the Du Pont Company of Canada spoke on "Statistical applications in the plant and laboratory." The speaker discussed some aspects of the problem of applying statistics in a continuous chemical (nylon) process. The meeting was well attended and the lively question and answer period certainly indicated that Mr. Eastcott's talk was most informative.

NEW HAVEN At the Winchester Clubhouse of Olin-Mathieson Chemical Corp., a panel discussion was held with our three experts O. H. Somers, R. J. D. Gillies, and W. Parker, answering questions on quality control problems presented by those members in attendance. Messrs. Somers, Gillies, and Parker offered suggestions in the practical applica-

tion of QC techniques and gave some answers to problems that had no theoretical solution. George S. Georgis did an admirable job in moderating the session and keeping the discussion on an even keel.

NORTHEASTERN INDIANA Our December meeting was held in the facilities of Indiana Technical College, Fort Wayne. The meeting was in conjunction with the Institute of Radio Engineers. M. E. King, a key official with the Bureau of Ordnance, Navy Department, was the speaker of the evening. Mr. King, spoke on the subject of "Evaluation of contractor's inspection." Mr. King's talk emphasized the many problems and remedies relative to an inspection system which is essential to be acceptable to the Bureau. The talk was excellent and thoroughly enjoyed by all those who attended.

PITTSBURGH On Dec. 11 the section had a joint winter meeting with the Pittsburgh Chapter of the American Statistical Association. The speaker was Dr. Stanley Isaacson who was very well received in his talk on the "Applications of statistical decision theory to industrial problems." For many of these present it was the first opportunity to hear such a theoretical subject discussed on the application level. Dr. Isaacson is to be congratulated for his very fine presentation.

ST. LOUIS On Dec. 21, Frederick J. Verigan of Crown Cork & Seal Co., Inc., Baltimore, spoke to the section on "Organizing and administration of quality control products" which was most enlightening. He explained how quality control is set up in the parent plant so the number of divisions under it will function properly.

A spring training course in intermediate quality control is again being held every Friday evening at the Melbourne Hotel and will continue through Apr. 5.

SALT LAKE CITY Following a dinner at the University of Utah, members and guests assembled to hear Loren Straw of United Air Lines speak on "Applications of statistical quality control to determine operating performance goals." Mr. Straw's address included the presentation of his data to management and the methods developed by United Air Lines in graphic presentation.

The equipment used in the graphic presentation attracted considerable attention and the questions asked regarding the methods of presenting data to management attested to the interest in the subject.

SAN DIEGO Larry Lawson, general supervisor of tooling inspection at Convair in San Diego, presented a most interesting discussion on optical tooling. He discussed the rigid tolerances required on high speed military and commercial airframes and how the advent of modern optical tooling has resulted in accuracy never before achieved. A film "The invisible line" was also shown. The meeting followed a pit barbecued beef dinner at the North Park Club.

SCRANTON -WILKES-BARRE The December meeting, held on the 17th, had increased attendance over our previous meetings. Prof. Clifton Anderson spoke on "Measures of dispersion" and his talk included most every measure applicable from the basic formulas to modern industrial interpretation. After the talk a question and answer period was enjoyed by all present.

SOUTH BEND -MISHAWAKA An advanced course in statistical QC is being conducted by the section. The 13 week course covers statistical methods adaptable to the solution of industrial problems. Two 13 week basic courses are also being offered.

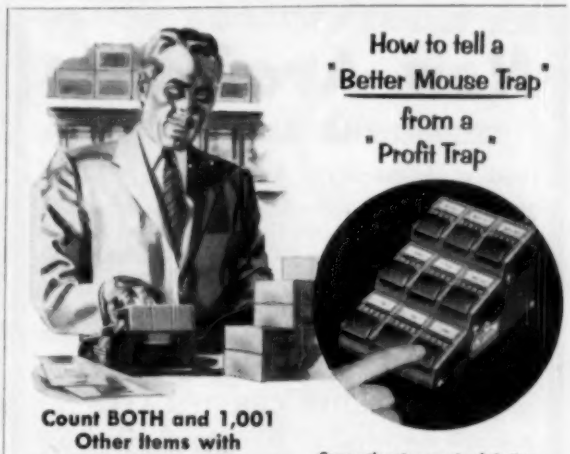
SOUTHERN CONNECTICUT On Dec. 12, sixty members of the section toured the Scoville Manufacturing Co. in Waterbury. The casting shop, tube extrusion department, and the continuous strip mill offered many points of interest to those attending.

A dinner, followed by a talk on vendor-consumer quality relations by host G. C. "Guns" Baldwin, assistant to the vice president, climaxed a very enjoyable day.

TOLEDO The members of the section and their wives enjoyed a gala Christmas party on Dec. 6 in the Elks Club. Following cocktails, dinner was served and was then followed by an amusing film. Dancing wound up this top social event of the season.

TORONTO At our last speaker meeting of 1956 the section was addressed by W. M. Bertholf, preparation engineer, Colorado Coal & Iron Corp., a prominent authority on bulk sampling.

Under the leadership of the forum committee, the executive of the Toronto section gathered for a social evening at the "Little Denmark" tavern in Toronto on Dec. 7. Nineteen members were in attendance. After an excellent roast beef dinner, several reels of motion pictures were shown.



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WACO Lt. Col. George W. McClughan, Air Force representative to AF Plant 66 at McGregor, Texas, was guest speaker at the Dec. 17 dinner meeting. Col. McClughan stated that quality control is a matter of teamwork between the contractor and the consumer and the result is a product of high quality standard and performance. He further stated that quality control is the thing that finally can make our country defensively sound.

WESTERN MASSACHUSETTS On Dec. 11 Joseph G. Griffin, superintendent of quality control, Westinghouse, Springfield, gave an enlightening talk on "Pre-testing packaged products for safe transit." Mr. Griffin, who is also section chairman, reviewed all the correlation tests made by the National Safe Transit organization and illustrated through movies as to how the tests were made. The advantages of securing National Safe Transit approval were reviewed with the group.

WINDSOR The family Christmas party on Dec. 19 proved to be extremely popular with the adults as well as with the children. Following the consumption of a delicious dinner, everyone joined in singing Christmas carols, ably led by Bert Underwood, accompanied at the piano by Bernie Stauth. For the next hour Mr. Kibble delighted one and all with his mastery of the art of sleight of hand. Then came the climax of the evening. Good old St. Nick appeared to present each child with a gift.

WORCESTER On Dec. 6 Dr. Milton Terry, a member of the Bell Telephone Laboratories technical staff at Murray Hill, N. J. spoke to the group on "Life testing." Dr. Terry did an excellent job of explaining a very technical subject. This was a very well attended meeting with a "Smorgasbord" dinner served at the Svea Grill Club.

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". . . CAST THEIR SHADOWS BEFORE"

ALLENTOWN -BETHLEHEM The March meeting will be held on the thirteenth at Walp's Restaurant, Allentown. Dr. Acheson J. Duncan of Johns Hopkins University will be the featured speaker. His address will be entitled "The design of control charts."

BUZZARDS BAY James A. Mitchell of the Tennessee Eastman Co., Kingsport, Tenn., will be main speaker at the Mar. 12 meeting to be held at the Continental Screw Co. in New Bedford. His topic will be "Control of the accuracy and precision of industrial tests and analyses."

CHICAGO Statistical quality control aspects of the consumer-supplier relationship will be explored by speakers at the Feb. 20 meeting to be held at the Furniture Club. Program chairman Eugene Freund, of the Victor Adding Machine Co., has chosen the topic "A method for rating suppliers quality" as his personal contribution. There will also be two other speakers whose names will be announced shortly. Starting at 4 pm this program will further include a social hour and dinner.

The basic educational program will feature a speaker on statistical sampling at its Mar. 5 meeting. This program is held at Roosevelt University, starting at 7:30 pm.

Roland Frakes, of the Du Pont Co., will lead the Mar. 19 meeting of the Calumet sub-group in a panel discussion covering statistical quality control in laboratory and production facilities of continuous process industries. Starting at 6:30 pm, this meeting will be held at Joe's Thornbridge Inn, Thornton, Ill.

CINCINNATI The Feb. 20 meeting will be long remembered by those who attend as it will be a plant tour at the General Electric Jet Engine plant in Evendale. Dinner will be served at the plant before the tour.

In connection with the observance of National Engineers' Week, the section will participate in a joint meeting with all of the Affiliated Technical and Scientific Societies Council of Cincinnati.

The February meeting of the Hamilton-Middletown subsection will feature Richard Hausman of the Ford Motor Co., Cincinnati, on the topic "Machine tolerance capabilities." The meeting will be held at the Whirlpool-Seeger Corp.

The March 13th meeting will begin with a dinner at the Manchester Hotel in Middletown and will be followed by a tour of the Armco Steel Co.

CLEVELAND "Statistics—a tool of management" will be the theme of the Mar. 1 meeting. R. B. Shartle is the speaker who will discuss this subject. Mr. Shartle is from the Standard Register Co. This meeting will be held at the Cleveland Engineering Society.

COLUMBUS The section will hold their annual management dinner meeting on Feb. 20 at 6:30 pm at the Deshler-Hilton Hotel. Paul Roberts, Gruen Watch Co., junior past president of the Society,

will be the guest speaker. Reservations can be made with any section officer.

CORNING -ELMIRA One of the major hi-lites of the year's program will be the dinner-meeting to be held at the Baron-Steuben Hotel, Corning, on Mar. 12. The speaker for the evening will be L. A. Seder, QC consultant from Malden, Mass., who will speak on "Who is responsible for quality." This will be a repeat performance for Mr. Seder, whose talk to the section last year was very well received.

DALLAS -FORT WORTH On Mar. 14 Dr. Carl A. Bennett, manager of operations research and synthesis, General Electric Co., Richland, Wash., will speak on "Statistical approach in chemical processing." The meeting will be held at Arlington State College in the new Engineering Building.

DAYTON The section will have its second annual all-day conference on Mar. 2. Arrangements for this conference are now complete. It will be held in two sessions at the Biltmore Hotel. Many top notch speakers from throughout the country have been obtained for this conference.

DELAWARE A very interesting meeting is scheduled for Mar. 7 consisting of a tour of the International Latex Corporation's plant at Dover, with an exposition of their inspection and quality control procedures. The tour of the plant will start at 2 pm following which the meeting will convene for dinner. After dinner representatives of the International Latex quality control team will discuss their procedures briefly, allowing ample time for an interesting question and answer period.

ERIE Our February meeting will be a joint session with the Erie chapter of AIIE. This was originally slated for Jan. 16th. James T. Lincoln of Lincoln Electric, Cleveland, will speak on Incentive management." February 28 is the proposed date.

On Mar. 13 our annual management night dinner meeting will feature Carlisle K. Milner, vice president of Ohio Bell Telephone, Cleveland. Mr. Milner's topic will be "Quality control in administration." Mr. Milner graciously consented to take the podium when Burt Seipel could not make an Erie appearance.

GEORGIA The Atlanta group will meet on Tuesday, Feb. 26, in Room 1833 of the Hurt Building in Atlanta, for an address by Harry O. Fewox, staff statistician, Southern Bell Telephone & Telegraph Co., on the subject "Customer acceptance measurement in the telephone industry."

HARRISBURG The section has been very fortunate in obtaining for the Mar. 6 meeting Miss Bonnie Small of the Western Electric Co., Allentown. Miss Small is one of the most widely recognized persons in the field of training in QC and will discuss "A practical training program for quality control people."

KANSAS CITY The Mar. 13 meeting will be held at the Blue Hills Restaurant. Following the 6 pm educational clinic and the dinner at 7 pm, H. R. Harrison, QC manager, Collins Radio Co., Cedar Rapids, Iowa, will be the guest speaker. Mr. Harrison's subject will be "What statistical quality control has done for Collins Radio."

LOUISVILLE Dr. Irving Burr, professor of mathematics and research associate in the statistical laboratory at Purdue University, will speak Mar. 11 on "Multiple correlation." Dr. Burr is author of *Engineering Statistics and Quality Control.*

MICHIGAN The Wolverine subsection will conduct a "Ladies Night" session next Mar. 1 at Post Cereals Club House in Battle Creek. Edward P. Lee of General Foods Corp., White Plains, N. Y., will discuss "Problems of weights and measures." As an added attraction at this dinner meeting, Dr. E. C. Yehle will present "The distaff side of quality control."

Leonard A. Seder, quality control consultant of Malden, Mass., will speak on "New methods of process capability analysis" at the Fort Shelby Hotel in Detroit next Mar. 15. This session will be co-sponsored by the American Society of Tool Engineers. An advanced educational program will be conducted at 6:30 pm. Andrew A. Procassini, Argus Cameras, Inc., Ann Arbor, will speak on "Contingency tables." The section is expecting a large turn-out.

MID-HUDSON The March meeting will feature an address by F. J. Berkenkamp of General Electric, who will talk on "Quality costs—a powerful measurement and programming tool." The meeting will be held on Mar. 5 at Nick Beni's Anchor Inn in Poughkeepsie.

MINNESOTA Dr. Max Astrachan will address the section on the subject "Sampling by variables" at the next meeting to be held on Mar. 18.

MONTREAL On Feb. 20 the section will hold a dinner meeting in the C.P.R. Windsor Station—Allouette Room. Harold Dodge of the Bell

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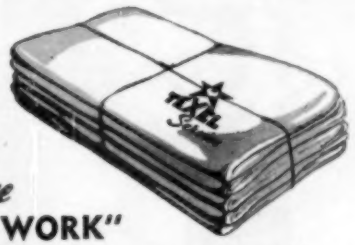

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MUNCIE Our regular monthly meeting will begin with a 6:30 pm dinner in the Pine Shelf Room of the L. A. Pittenger Student Center, Thursday night, Feb. 21. At 7:30 pm, J. A. Davies of General Electric's Owensboro, Kentucky Division will deliver an address entitled, "Variables sampling without calculation."

NEW HAVEN On Mar. 19 J. W. Sullivan of the American Steel and Iron Institute will speak on "Statistical methods in steel technology." The meeting will be held in the Chamber of Commerce auditorium in downtown New Haven.

NORTHEASTERN INDIANA Dr. Robert Lussner, Redstone Arsenal, Huntsville, Ala., will be speaker at the Mar. 14 meeting of the section to be held in the Chamber of Commerce, Fort Wayne. Dr. Lussner's subject will be "The notorious unreliability of complex equipment and reliability demonstration." The section will be meeting jointly with the American Institute of Industrial Engineers.

ROCHESTER On Mar. 5 the gaging and measurement sub-group will meet with Leonard Myers, gage engineer, Camera Works, Eastman Kodak Co. Mr. Myers' topic will be "Standards and Specifications—useful tools in gage control programs."

The Mar. 19 meeting will be addressed by Warren R. Purcell of Rath & Strong, Inc., Boston. The subject of Mr. Purcell's address will be "Quality control in continuous processes."

SAN DIEGO The Mar. 11 meeting promises to be a highlight of the year. This meeting is a panel meeting moderated by J. Dagan, chief of quality control, Rohr Aircraft Corp., Chula Vista, Calif. Mr. Dagan's panel members are: L. B. Allen, Solar, San Diego; L. M. Limbach, Ryan Aeronautical, San Diego; A. P. Higgins, Convair, San Diego; and B. F. Raynes, Rohr Aircraft. The topic will be "What management expects from quality control." This meeting will be a dinner meeting at the North Park Club.

SCRANTON -WILKES-BARRE We are looking forward to our February meeting. This meeting will be centered on the subject "Quality control in the aircraft industry."

SOUTH BEND -MISHAWAKA Dale L. Lobsinger, President of the Society, will speak to the section on Mar. 21. His title, "QC for the air lines," reflects his job in performance and controls of United Air Lines. Arrangements have been made to make the honored guest feel at home.

TOLEDO The section has scheduled a joint meeting with ASTE, AIIE, and ASME at the Maumee River Yacht Club in February. This is another outstanding meeting of the section as these joint meetings are well planned. Nevin Bean from Ford's engineering division in Detroit is scheduled to give a first hand report on "Industrialization in Russia" as a result of his tour of that country.

Is Your Section "High" or "Low" Man on the Totem Pole?

1. Albany
2. Albuquerque
3. Baltimore
4. Baton Rouge-New Orleans
5. Birmingham
6. Burlington
7. Buzzards Bay
8. Central Illinois
9. Chattanooga
10. Chicago
11. Cincinnati
12. Cleveland
13. Columbus
14. Corning-Elmira
15. Cumberland
16. Dallas-Fort Worth
17. Danville-Sunbury
18. Dayton
19. Denver
20. Erie
21. Georgia
22. Grand Rapids
23. Greater Muskegon
24. Harrisburg
25. Hartford
26. Indianapolis
27. Iowa
28. Japan
29. Kansas City
30. Los Angeles
31. Louisville
32. Memphis
33. Metropolitan
34. Mexico City
35. Michigan
36. Mid Hudson
37. Minnesota
38. Montreal
39. Muncie
40. New Haven
41. Parkersburg
42. Philadelphia
43. Pittsburgh
44. Pittsfield
45. Portland
46. Rhode Island
47. Richmond, Va.
48. Rockford
49. St. Catharines
50. St. Louis
51. Salt Lake City
52. San Antonio
53. San Bernardino
54. San Diego
55. San Francisco
56. Scranton-Wilkes-Barre
57. South Texas
58. Syracuse
59. Toronto
60. Tucson
61. Utica
62. Waco
63. Washington
64. Western Massachusetts
65. Western Ontario
66. Wichita
67. Windsor
68. Youngstown

At the left are listed in alphabetical order all of the sections whose membership performance is significantly better or worse than the average. Using each section's membership as reported to the National Headquarters office by Jan. 24, 1957, performance was calculated as a percentage of last year's membership at June 30.

The sections listed at the left are those that fall above or below the 95 percent confidence band. The overall average for the Society this month is 95.0 percent. Again worthy of special mention is Harrisburg with a growth of 182 percent; low man on the totem pole was still San Antonio with 34.4 percent.

Also of interest is the major reason for the changes between this list and last months'. The society average increased from 91.6 percent on Dec. 24; consequently sections with insufficient or no increase in membership during the two months covered, now appear on the low side.

Performance significantly better than average:	Performance significantly poorer than average:
2, 11, 13, 14, 15, 18, 20, 23, 24, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 47, 48, 50, 54, 55, 57, 59, 63, 64, 65.	1, 3, 4, 5, 6, 7, 8, 9, 10, 12, 16, 17, 19, 21, 22, 25, 26, 27, 35, 37, 39, 41, 44, 45, 46, 49, 51, 52, 53, 56, 58, 60, 61, 62, 66, 67, 68.

TORONTO A visit to the environmental test laboratory at P.S.C. Applied Research Co. in Toronto has been planned for Feb. 27. Members will have the opportunity of seeing in action the advanced equipment necessary for testing components for use in high speed aircraft and other rigorous applications.

WESTERN MASSACHUSETTS On Tuesday, Mar. 12, David N. Smith, manager of research, Jones and Lamson Machine Co., will lecture on "Automatic regulation of machine tools on a statistical control basis."

WORCESTER On Feb. 21 Warren Purcell will speak on "The pursuit of facts." The time and place of this meeting will be announced at a later date.

Watch for a special announcement of an education series. The instructor for this series will be Axel Sternloff of the American Steel and Wire Co.

"AS YE SOW . . ."

ALBUQUERQUE The Community College of the University of New Mexico, in cooperation with the section, is offering the second semester of statistical quality control. This advanced course will cover such subjects as: frequency distributions including normal, Poisson, and hypergeometric; acceptance sampling by attributes and variables; tests of significance and variance; regression analysis; correlation; and analysis of variance. The course will be taught by J. M. Wiesen and G. J. Lombardi, both of Sandia Corp.

ALLENTOWN -BETHLEHEM The third section sponsored course of this season, "An introduction to the design of experiments," will be presented Monday evenings, Mar. 18-May 6. Classes will be held at Muhlenberg College, Allentown, Pa.

BALTIMORE Several conferences have been planned on the subject of advanced use of statistical methods. The conferences will be under the direction of Dr. A. J. Duncan, of the Johns Hopkins University.

BOSTON Plans are nearly set on a repeat of the course given particularly for engineers last spring. This brief course seemed to fill one of the needs long realized by members in this area in providing an introduction to the possibilities of industrial statistics in solving engineering and technical problems for engineers not primarily concerned with QC problems. Details have been mailed to members and other engineering groups. In view of the large attendance last year, early registration is advisable.

CINCINNATI The Hamilton-Middletown subsection will repeat the very popular introductory course on "Quality control techniques" in February. R. J. Hausman will be the instructor. In addition, due to numerous requests, an advanced course will be offered with DuWayne Carlson of the Gardner Board and Carton Co. of Middletown as the leader.

DALLAS -FORT WORTH The second annual quality control symposium will be held on Mar. 16 at Southern Methodist University. Men who have distinguished themselves in the various industrial applications of QC will be the speakers in the all-day symposium. The arrangements for the program are under the direction of Dr. John L. Wortham, Texas Christian University, Fort Worth. All inquiries regarding the symposium should be sent to Dr. Wortham, or to Roger W. Anderson, Texas Instruments, Dallas.

INDIANAPOLIS The section is sponsoring two courses in adult education at Indiana University in Bloomington. Member Al Gardner is to be commended for making the arrangements. Mel Handerson and Russ Kennedy are the instructors.

MILWAUKEE Our annual clinic is scheduled for Mar. 15. The 12 sessions are in four concurrent series; brewing, administrative applications, and two general groups. Special attention is being paid to getting those new to SQC started and to interest some of the smaller firms. The problems of machine capability, simplifying SQC paperwork, pilot plant problems, inventory control, operations research, what the QC man should know about linear programing, designing experiments, are but a few of the topics.

More information may be obtained by writing to P. O. Box 1204, Milwaukee 1, Wis.

FIFTH PACIFIC NORTHWEST QC CONFERENCE The Portland and Seattle sections are co-sponsors of the Fifth Pacific Northwest Quality Control Conference. The two-day conference will be held at the Columbia Athletic Club, 614 W. 11th Street, Portland, Ore., Mar. 22-23.

SCRANTON -WILKES-BARRE The second semester of statistical quality control at Penn State University extension will include the second part of Eugene Grant's book and will be taught by Carl Larson, manager of quality control at

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TORONTO Twelve speakers from all parts of the United States and Canada will be at Hart House, University of Toronto, on Saturday, Mar. 9, to take part in the Fourth Annual all-day forum sponsored by the section. Participation in the forum by over 300 members from neighboring areas is anticipated. The program will be presented as three concurrent sessions followed by an evening banquet and will offer much to: executives and managers; those using quality control but not familiar with statistical methods; those acquainted with statistical quality control but new to the all-day forum; past registrants of the all-day quality control forums. Reprints of the talks will be available to all registrants. For further information, please contact Treasurer, All-day Forum, Box 22, Station R, Toronto 17, Ont.

GEORGIA INSTITUTE OF TECHNOLOGY A basic course in "Quality control by statistical methods" will be conducted at the Georgia Institute of Technology, Mar. 12-20; for application or further information write: Coordinator, Short Courses and Conferences, Georgia Institute of Technology, Atlanta, Ga.

SEVENTH REGION IRE CONFERENCE APRIL 24-26 The Institute of Radio Engineers will hold their seventh region conference in San Diego, in the Conference and Federal Buildings in Balboa Park, Apr. 24-26. Many dozens of outstanding technical papers of broad interest will be presented throughout the entire show which has the theme, "The Theme of the Age."

There will be opportunities to see the facilities of many of the area's leading electronic manufacturers to observe new production methods and to discuss common engineering and design problems.

There will be over 200 interesting and valuable technical exhibits of major electronics manufacturers.

There is a very adequate and interesting program for the ladies which includes a fashion show and special luncheon.

On the entertainment side—bull fights, Jai Alai, and horse racing in old Mexico, just to mention a few.

All mail regarding the conference should be addressed to IRE Seventh Region Conference, U. S. Grant Hotel, San Diego, Calif.

STANFORD UNIVERSITY TUITION SCHOLARSHIP The San Francisco Bay Area section has joined the department of industrial engineering of Stanford in sponsoring a Stanford University tuition scholarship. The scholarship will be for the 1958-59 school year and will be given to a selected graduate student for study and research in statistical quality control.

The Bay Area section is using funds accrued from previous annual Bay Area QC conferences which have been held at Stanford for the past three years. If sufficient conference funds continue to accrue, it is planned to make this a continuing scholarship.

SOUTHERN REGIONAL SUMMER SESSIONS IN STATISTICS The fourth Southern Regional graduate summer session in statistics will be held from June 12 through July 20 at the Virginia Polytechnic Institute, Blacksburg, Va. Each annual summer session offers courses carrying approximately five quarter hours of graduate credit. The program may be entered at any session and consecutive courses will be offered in successive summers.

The sessions will be of particular interest to; research and professional workers who want intensive instruction in basic statistical concepts and who wish to learn modern statistical methodology, teachers of elementary statistical courses who want some formal training in modern statistics, prospective candidates for graduate degrees in statistics, graduate students in other fields who desire supporting work in statistics, and professional statisticians who wish to keep informed of advanced specialized theory and methods.

The total tuition fee will be $38.00 for the six-week term. Doctoral courtesy will be offered to those holding doctoral degrees. Living and other expenses at the Virginia Polytechnic Institute are reasonable. The Virginia Polytechnic Institute is located at Blacksburg, Va., on the scenic Alleghany Mountain plateau, 2,100 feet above sea level. The summer climate is delightful.

Inquiries should be addressed to Boyd Harshbarger, Head, Department of Statistics, Virginia Polytechnic Institute, Blacksburg, Va.

UNIVERSITY OF ILLINOIS, QC SHORT COURSE The tenth annual short course in "Quality control by statistical methods" will be offered by the College of Engineering, University of Illinois with the cooperation of the division of engineering extension at Urbana from Mar. 11-Mar. 21.

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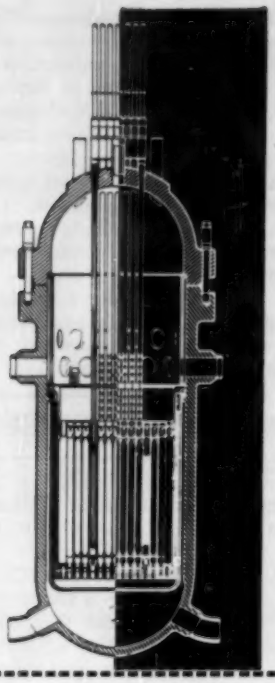

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  - ☐ Non-destructive Testing
  - ☐ QUALITY CONTROL
  - ☐ Mechanisms
  - ☐ Pumps & Valves
  - ☐ Non-Destructive Testing
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  - ☐ Pumps & Valves
  - ☐ Instrumentation
  - ☐ Piping
  - ☐ Welding
  - ☐ Heavy Fabrication
  - ☐ Machine & Tool Design
  - ☐ Machining & Heat Treating

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"FOR MANY ARE CALLED, BUT FEW ARE . . ."

PORTLAND In the month of November, Ray Vanderzanden replaced Robert Webster as section chairman. Mr. Webster has been transferred to Seattle and will be inactive in the section. Mr. Vanderzanden had previously been vice chairman and program chairman.

DIVISIONS

Chemical Division

An all-day symposium on statistical methods in the chemical industry, sponsored by the Metropolitan section and the Division in cooperation with the Stevens Institute of Technology, was held at the Institute on Jan. 12. The luncheon speaker was Dr. G. E. P. Box.

The Division is pleased to note that Dr. Box, formerly of Imperial Chemicals, Ltd., is now associated with Princeton University as head of the statistical techniques group. Dr. Box is the originator of certain methods of statistical analysis commonly known as the "Box method" which has been the subject of a three-day course offered by the Division at Harvard Business School in May, at University of Michigan last October, and will be offered at Houston, Texas in mid-February. The course instructor, Dr. J. S. Hunter, has recently joined Dr. Box at Princeton, but continues his former connection with American Cyanamid Co. as a consultant.

The winter meeting of the Division Council was held in Houston, Texas.

The Division has arranged for a series of papers on the Box Technique and on the general subject of industrial experimentation to be presented at the Middle Atlantic Quality Control Conference.

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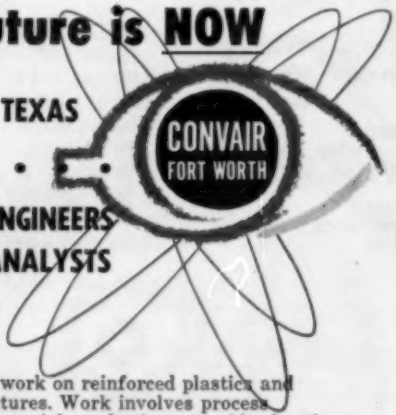

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SIGNIFICANT DIFFERENCES

Boston section member Howard Keefe, for many years at the Riverside Press, has recently joined the quality control staff at the Hood Rubber Co., Watertown.

Howard Jones of the Bell Telephone Co., a member of the Chicago section, addressed a December meeting of the Chicago Technical Societies Council. His subject was "Administrative application of statistical quality control." This meeting was arranged by another section member, Harry E. Sagen, who serves as chairman of the Council's arrangements committee.

Relation of federal regulatory agencies to the quality control program for pharmaceutical production techniques was the topic for a talk delivered by Chicago section member L. Feiertag on Dec. 10 at a meeting of the Abbott Study Club at Abbott Laboratories.

A. R. Jackson participated on the teaching staff of a two-day seminar on "Inspection organization" held at the University of Wisconsin during December. Mr. Jackson, a member of the Chicago section, spoke on the topic "Better product quality with a universal quality index."

Cincinnati section chairman Dave Birch regretfully announced the transfer of Al Evans of the Monsanto Chemical Co. from Cincinnati to Springfield, Mass. Mr. Evans, January conference general chairman, has been a very active member of the section.

Recent organizational changes at the Corning Glass Works affected several Corning-Elmira section members. These included: Luther A. Ackerson, who transferred from QC supervisor of the components department to production foreman at the apparatus plant; Andrew Flood, QC analyst to production planning; Peter Wesel, process engineer to supervisor of mold equipment; Robert Lyons, QC supervisor to production foreman; and Eugene McIlwain, QC supervisor to process and quality engineer at the refractories plant.

Past chairman Ivan L. Lester of the Georgia section is now the works manager of the Georgia Kaolin Co. at Dry Branch, near Macon, Ga., but he continues his interest in quality control.

Roscoe M. Smith, director of quality control of the Ford Motor Co. in Dearborn, Mich., was honored at a retirement dinner by 400 of his co-workers and friends at the Latin Quarter in Detroit recently upon comple-

tion of his 40th year of service with Ford. Mr. Smith was a senior member of the Michigan section. He is succeeded by Lowell F. Sarnes.

Brent C. Jacob, Jr., chief industrial engineer, Crysler Div., Chrysler Corp., has recently taken a new position with Minneapolis-Moline. Mr. Jacob will be vice president in charge of manufacturing. Brent has been very active in the Michigan section as a committee chairman, speaker, and officer of the automotive section.

Dick Hoyt, arrangements chairman of the Mid-Hudson section, has been transferred by IBM to its San Jose location as technical assistant to the quality manager.

Sid Price, Harry Harding, and Dick Hulnick—members of the Mid-Hudson section—will present technical papers at the National I.R.E. conferences to be held in New York in March. Messrs. Price and Harding will discuss narrow limit gaging, a technique developed by them at IBM Military Products Div., Kingston. Mr. Hulnick's topic will be reliability of complex electronic equipment.

Dr. P. K. Leatherman, St. Louis section, spoke before a group of 50 at the Springfield-Decatur Society for Quality Control which hopes to become a subsection of the Central Illinois section. The meeting was held at the auditorium of the Sangamo Electric Co., Springfield, and his talk was entitled "Cookbook experiences in quality control."

Harold C. Hilditch has resigned his position as supervisor of statistical analysis with the lamp department of Canadian General Electric Co., Ltd., to join the firm of Deacon & Associates, Brantford, Ont. Mr. Hilditch, a member of the Toronto section, will be working with Allin P. Deacon on consulting work in SQC.

OUR FIRST STUDENT BRANCH

The first Student Branch of ASQC was formally organized in December of last year at City College of New York, under the sponsorship of the Metropolitan Section. Thirty students have joined the Society as Members-In-Training and have elected the following as officers:

Chairman:	Howard Kayton
Vice Chairman:	Manus Rabinowitz
Secretary:	Vivian Small
Treasurer:	Philip Lustig

Mr. Gerald Sandler, a member of the Metropolitan Section of ASQC and a member of the faculty of the College, has been appointed Counselor.

The students comprising our new Branch attend the Baruch School of Business of the College and are also organized as the Statistical Association of City College. The group meets once a week, on Thursdays, during the school year. Several photographs of the group show a number of the members, officers, and faculty, including Mr. Sandler, Dr. Herbert Arkin, Chairman of the Statistics Division of the School, and Mr. David Valinsky, also of the faculty and Vice Chairman of the Metropolitan Section.

Arrangements are being made for a Student Branch charter for this group, and for its presentation at a later date. The Officers and Directors of the Society congratulate the Metropolitan Section on establishing our first Student Branch and express appreciation for the efforts of Mr. F. B. May, Chairman, Mr. Valinsky, Mr. Sandler and Mr. Paul Chook, for carrying on this program. Best wishes go to the members of the new Branch and they may be sure that the Society is proud of their participation in ASQC activities. We hope, too, that this marks only the beginning of Student Branch activity throughout the country.

Here is a group picture of some members of the Branch. Holding the sign and fourth from the left is Howard Kayton, Chairman of the newly formed Student Branch. In the second row, extreme right, is Vice Chairman Manus Rabinowitz.

Dr. Herbert Arkin, Chairman of Statistics Division of the Baruch School of Business, City College of New York, is shown congratulating Chairman Howard Kayton. At Mr. Kayton's right is Gerald Sandler, ASQC Counselor for the Branch, and at the extreme right of the photo is David Valinsky, Vice Chairman of the Metropolitan Section.

This group picture includes (left to right) Student Members Mervyn Schactman, Gerald Fartah, Howard Kayton, Constantine Kollitus, Jack Friedland, Lawrence Mars, and Allan Dantowitz. Continuing are faculty members Gerald Sandler, Dr. Herbert Arkin, and David Valinsky.

Overheard at a local boys' club:
(five year old) Is Dad really boss around our house?
(eight year old) Certainly he is. Didn't you hear Mom say he was, and everything Mom says, goes!

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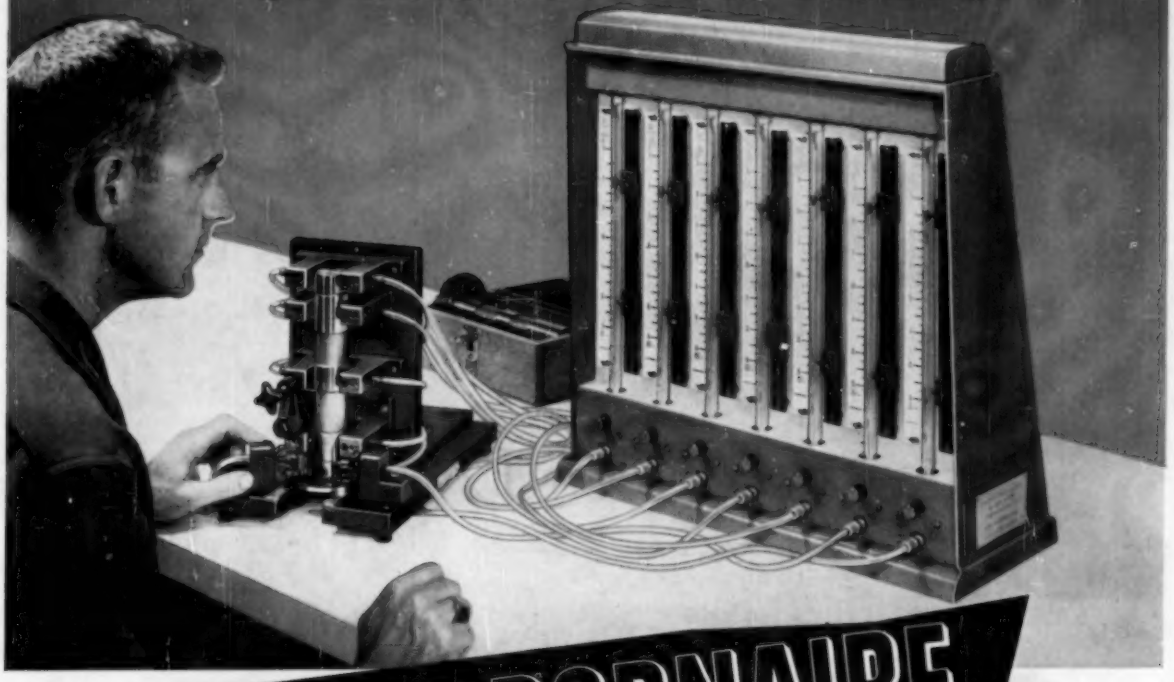

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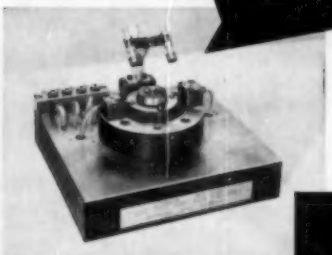

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